The agreement power of disagreement

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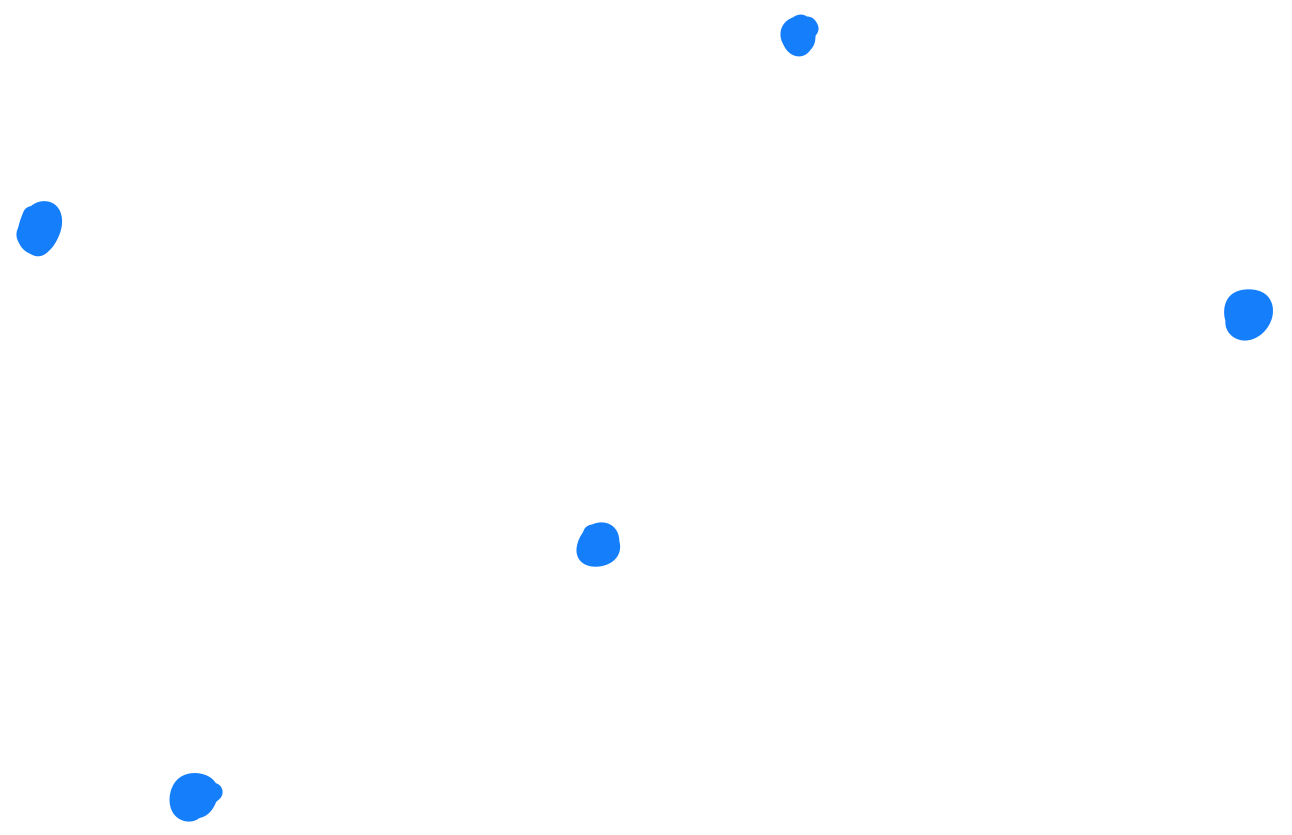
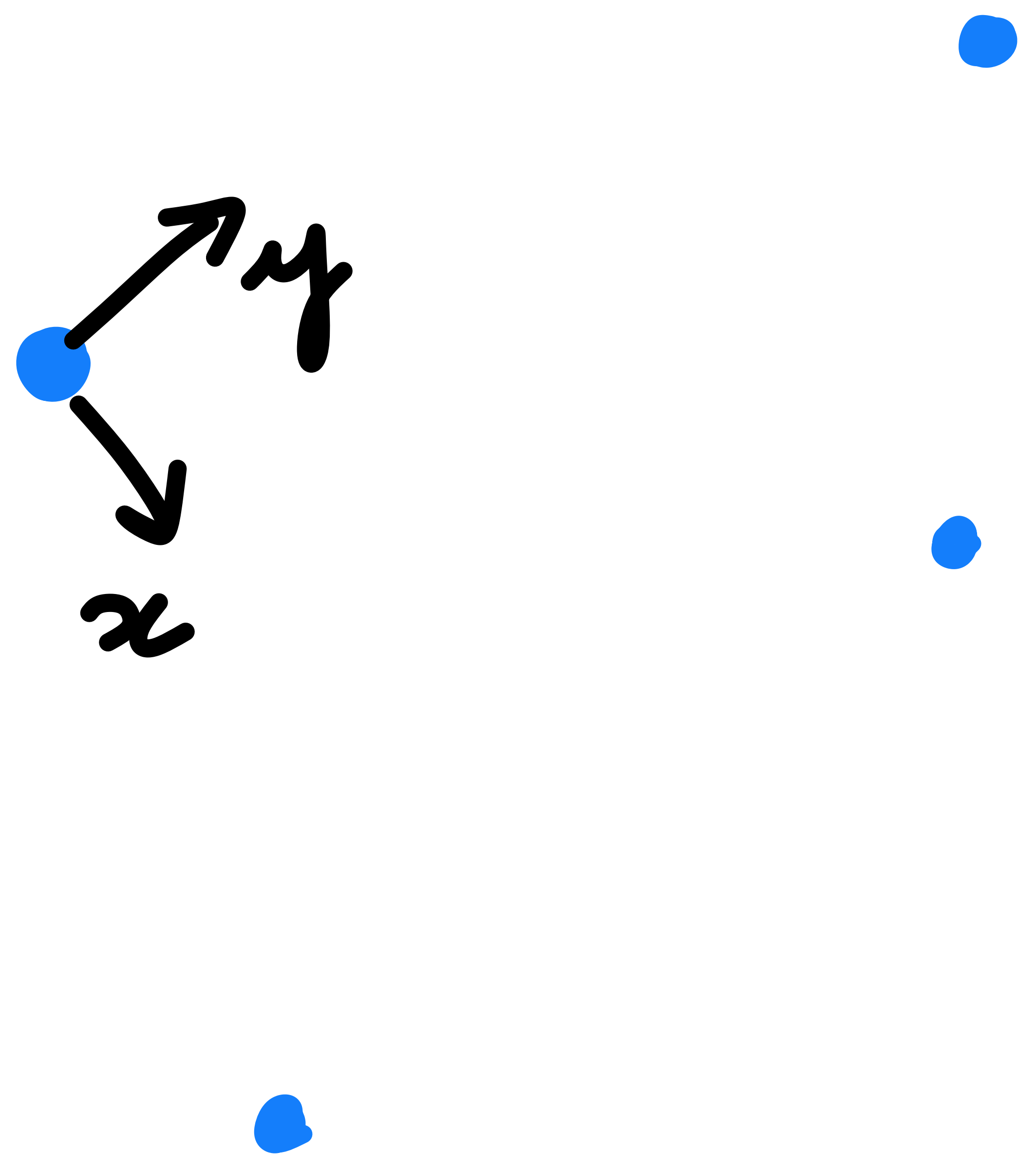
Mobile Autonomous Robots

- Anonymous
- Uniform
- Disoriented
- Silent
- Oblivious



Execution Cycle

- Look
- Compute
- Move



- Fully-Synchronous
- Semi-Synchronous

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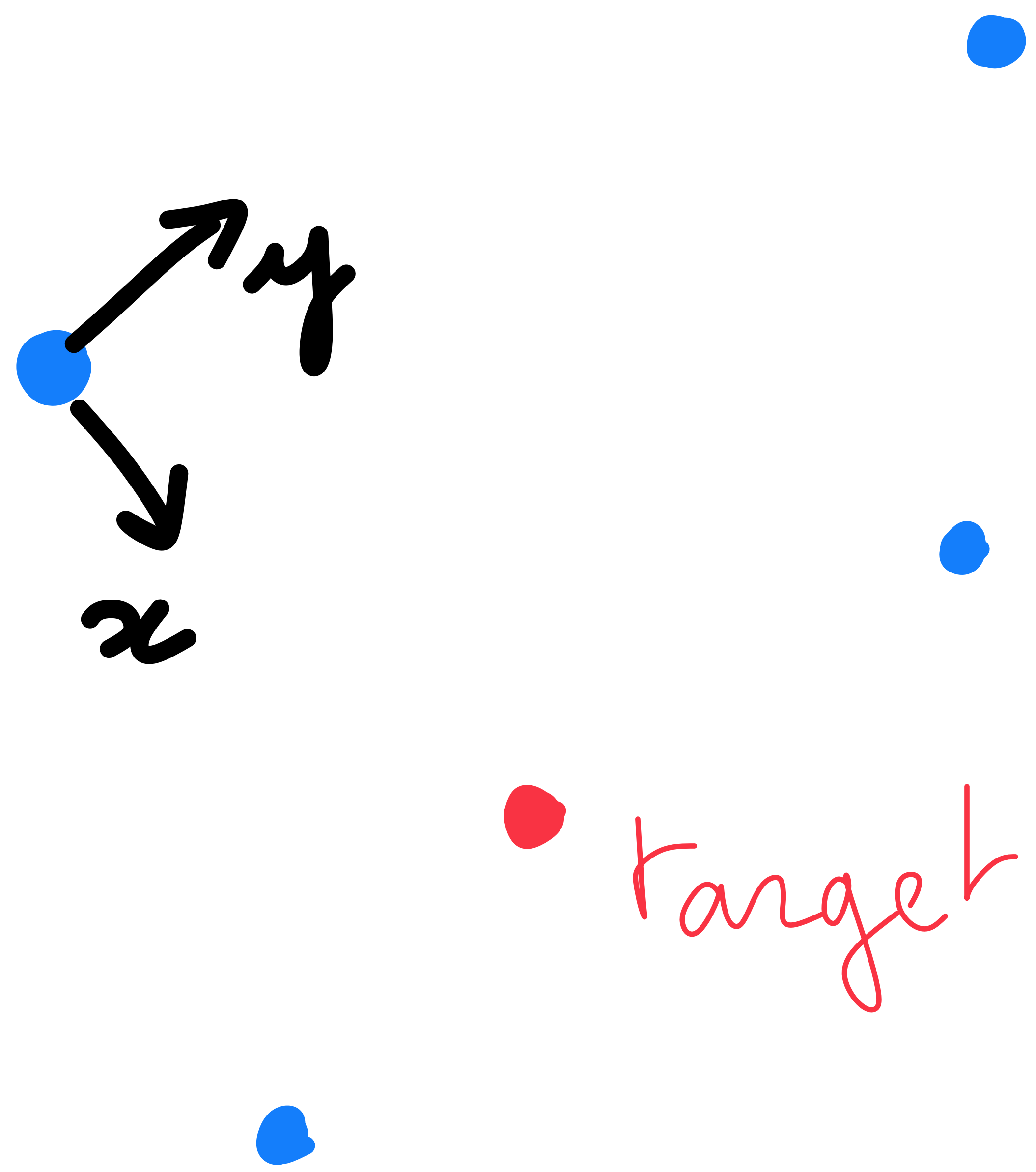

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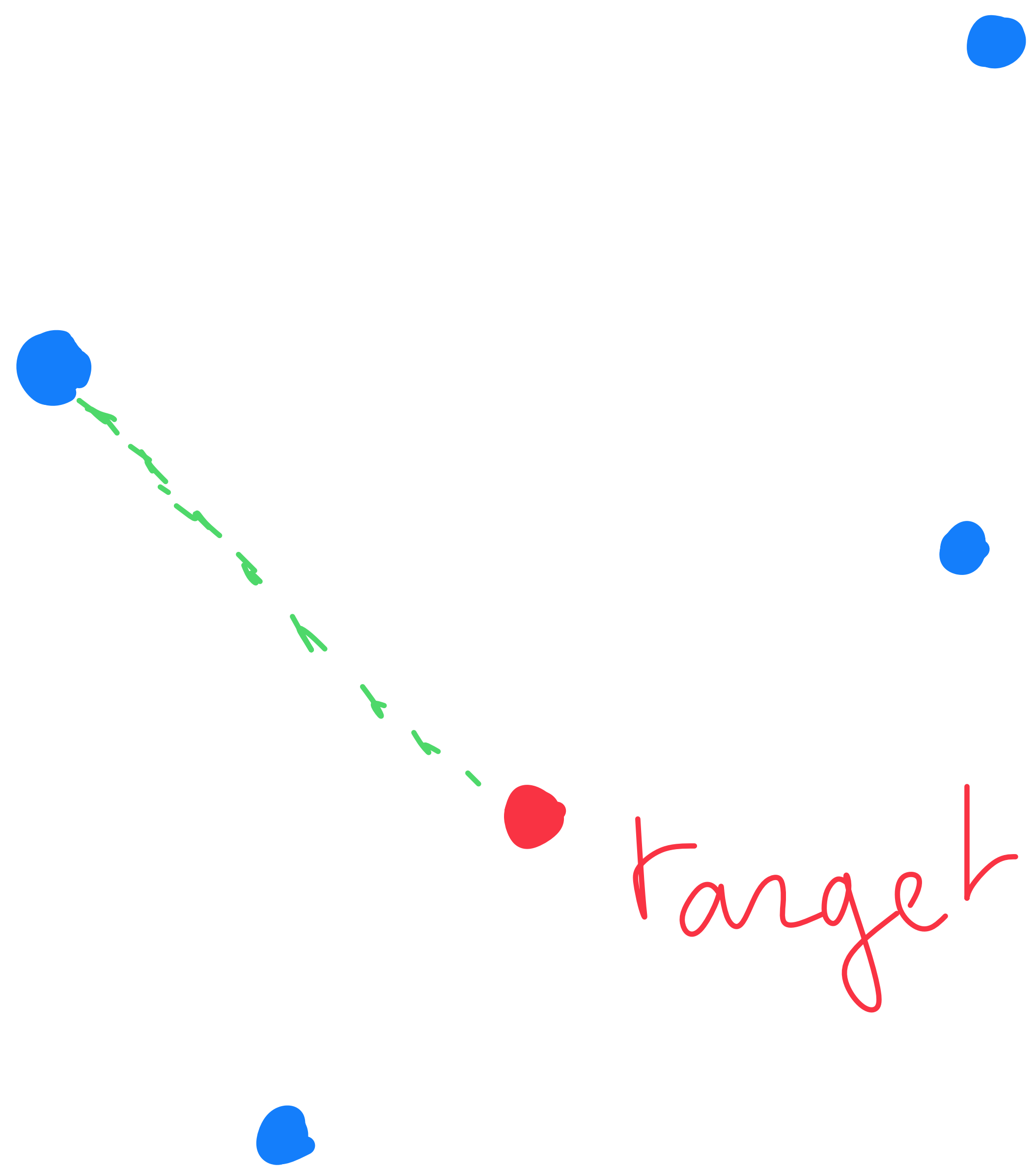


Execution Cycle

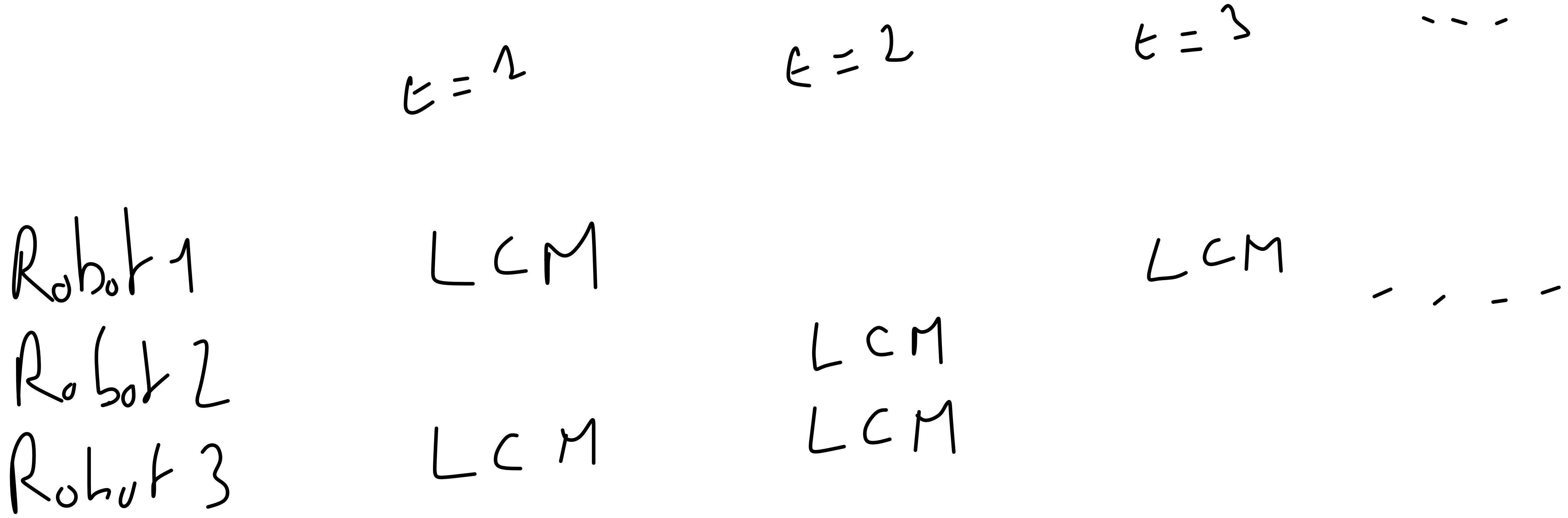
- Look
- Compute
- Move

- Fully-Synchronous
- Semi-Synchronous

	t = 1	t = 2	t = 3	...
Robot 1	LCM	LCM	LCM	...
Robot 2	LCM	LCM	LCM	
Robot 3	LCM	LCM	LCM	

Execution Cycle

- Look
- Compute
- Move



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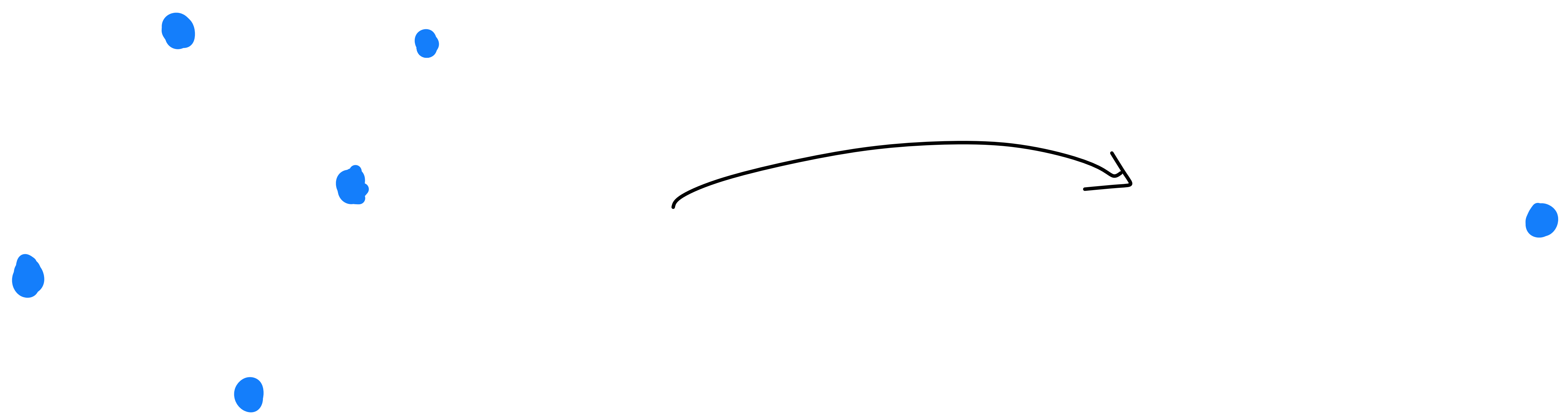
The Fundamental Problem of Gathering



Related Work

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FSYNC

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FSYNC

Solvable in FSYNC (Move to the middle)

The Impossibility of Rendezvous

[Suzuki & Yamashita 99]

Impossibility of rendezvous in SSYNC

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Impossibility of rendezvous in SSYNC

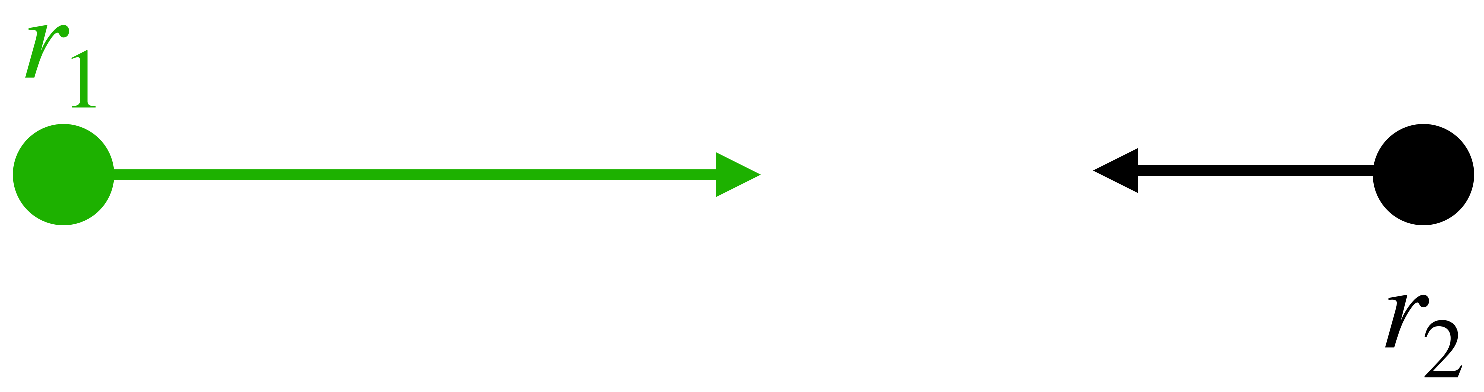
Assume an algorithm exists

There must exists a rule in the algorithm saying: « in view V, stay idle »

Both robots have view V, and stay Idle.

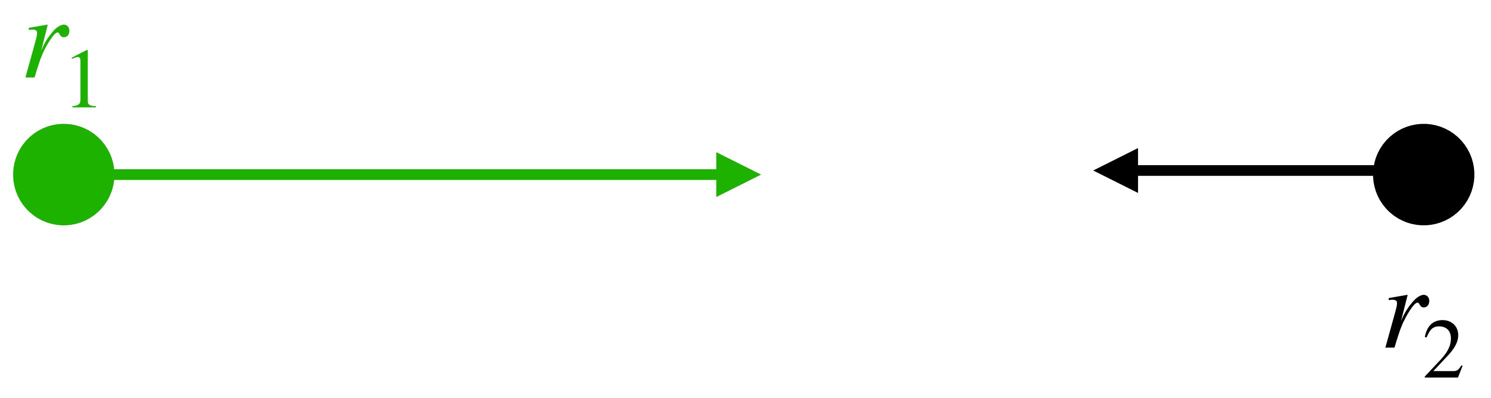
Rendezvous
Robots with different unit-distance

Notation



Notation

We have two robots r_1 and r_2 having unit distance $unit_1$ and $unit_2$



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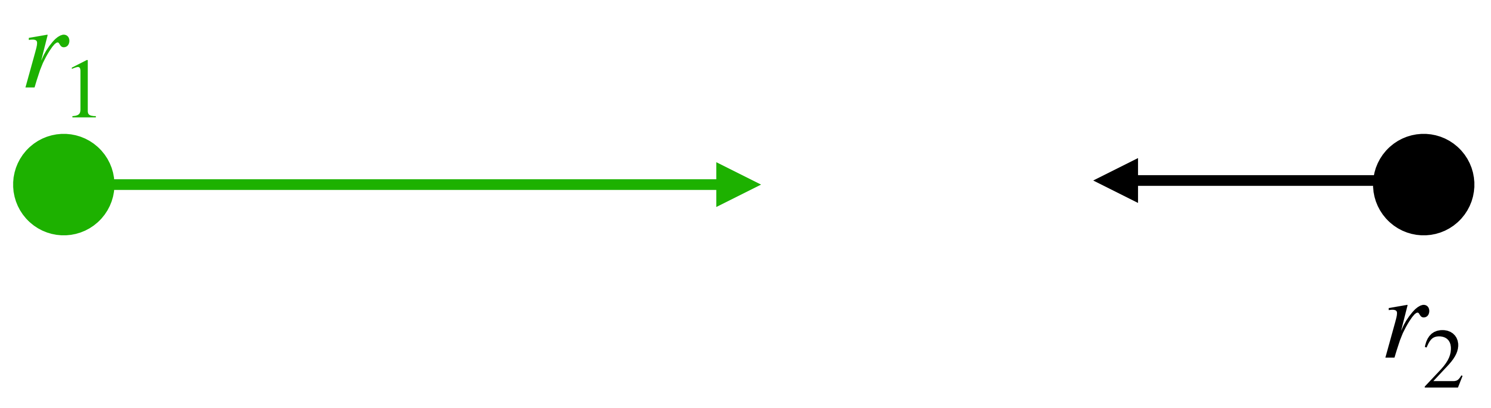

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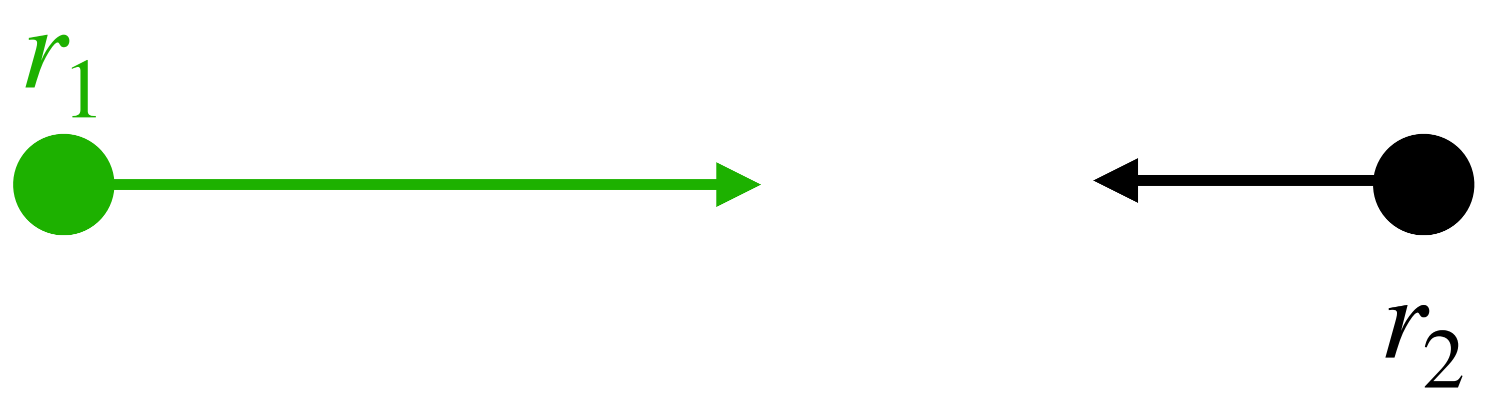
This implies that robots see the distance between them differently



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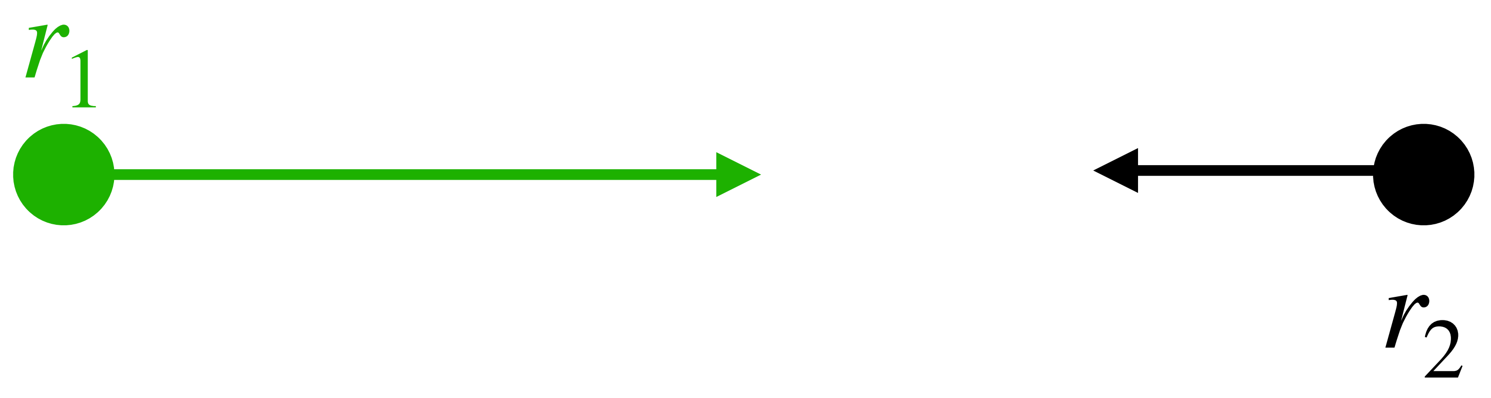

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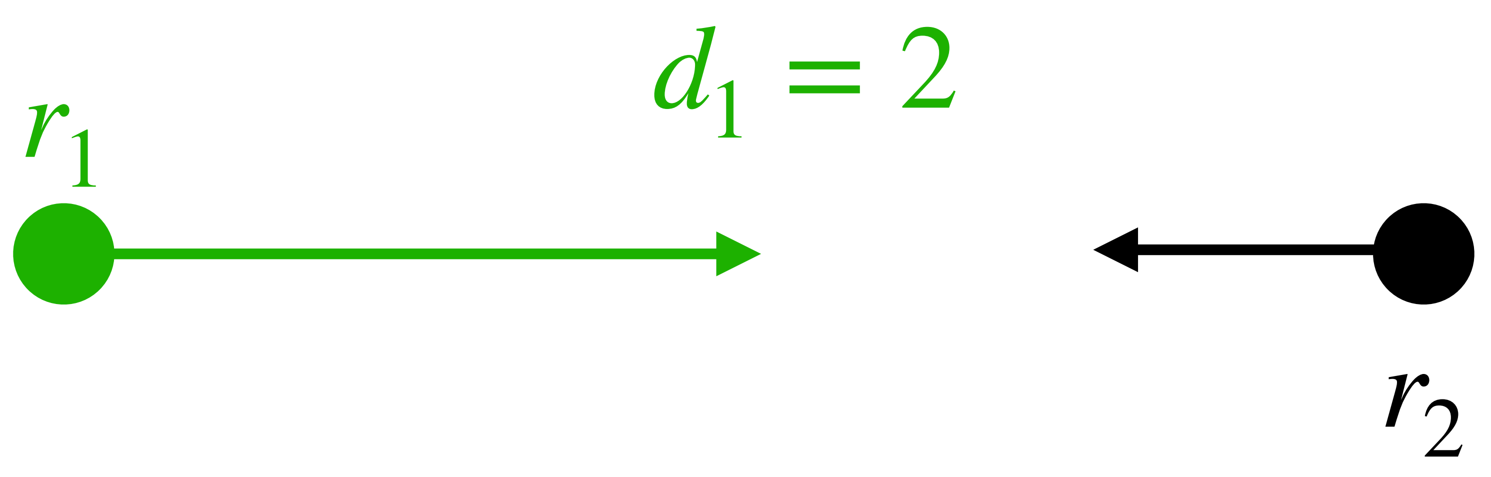

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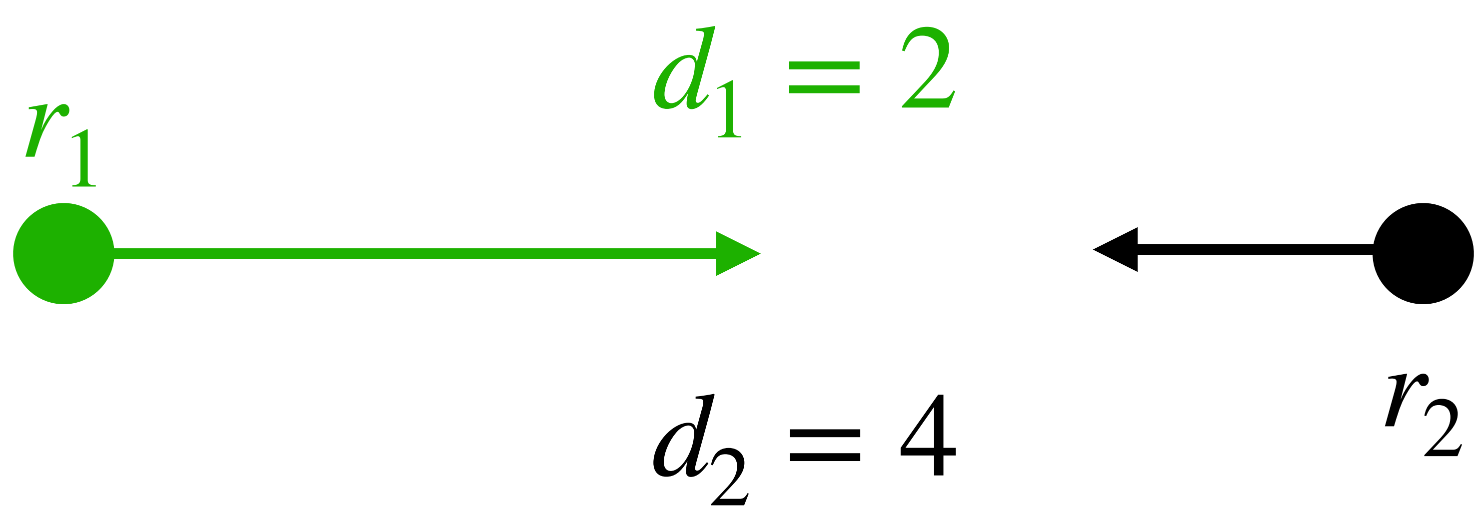
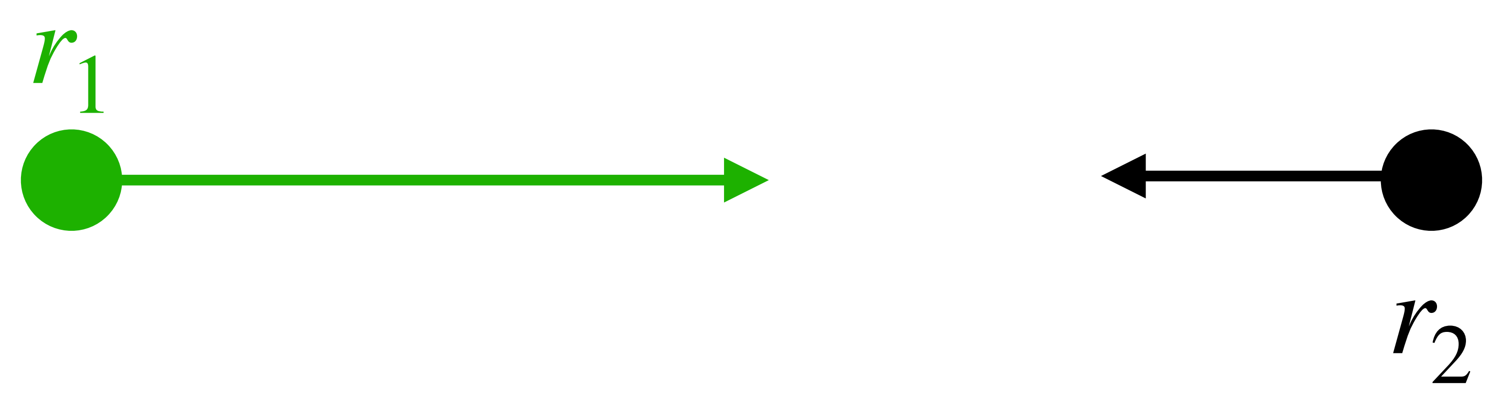


Rendezvous

$$\rho = 2$$

Rendezvous when $\rho = 2$



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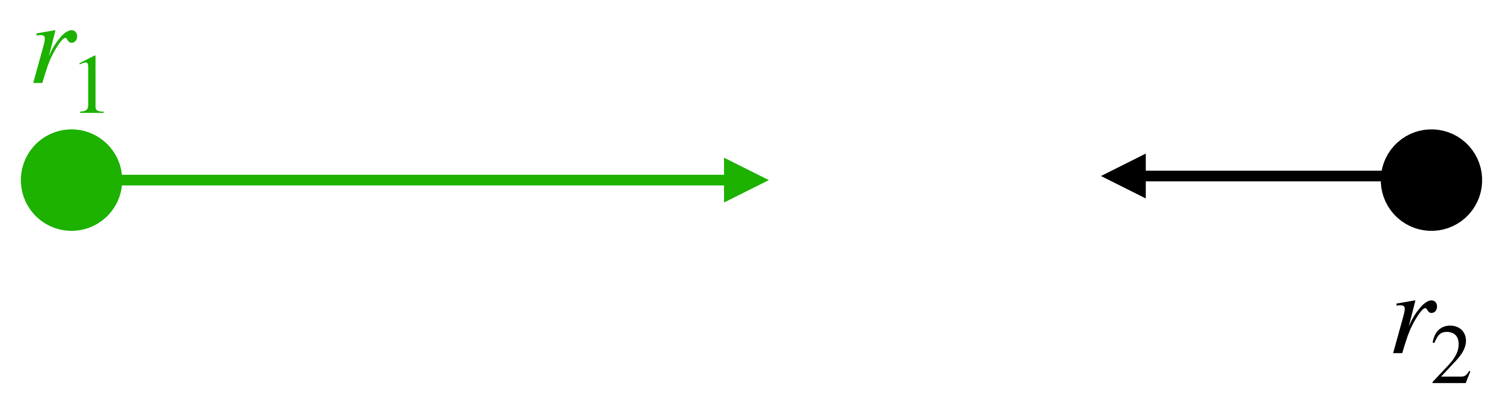

Definition:

We define ***the level of robot*** r_i

the unique number l_i

such that $d_i \in [2^{-l_i}, 2^{-l_i+1})$

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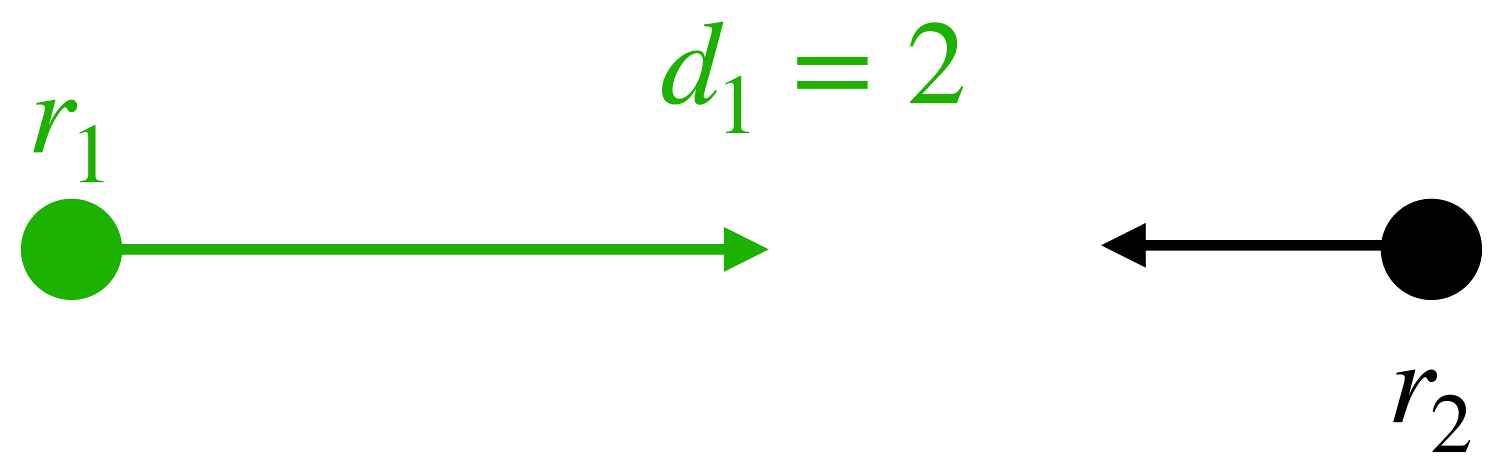

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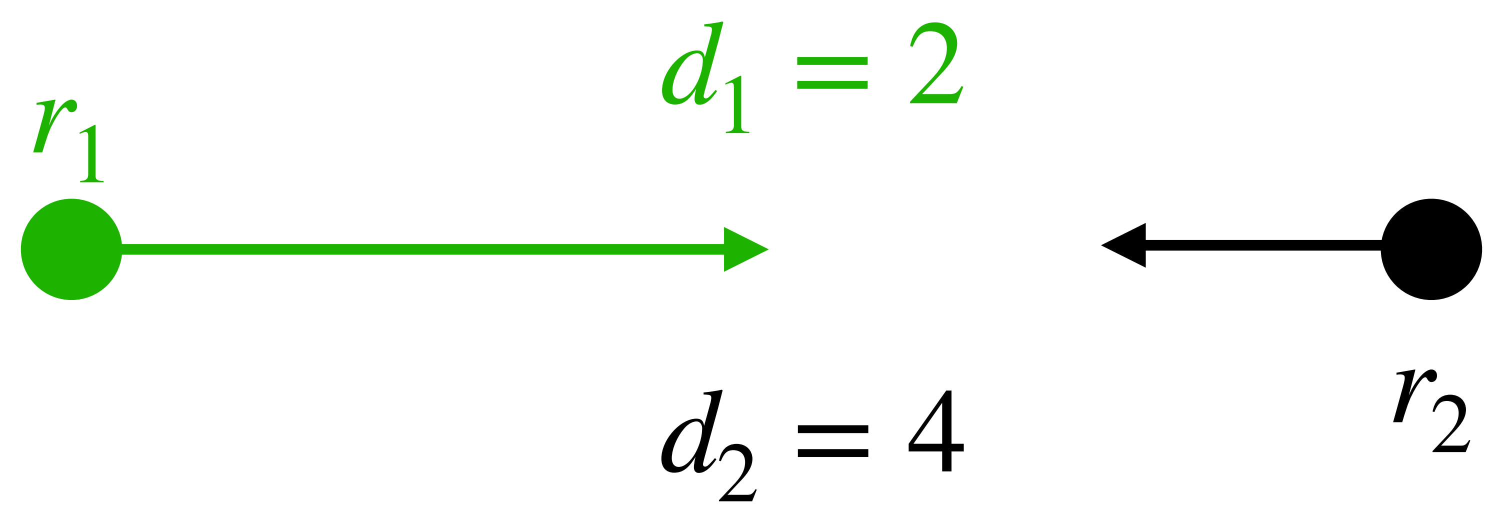

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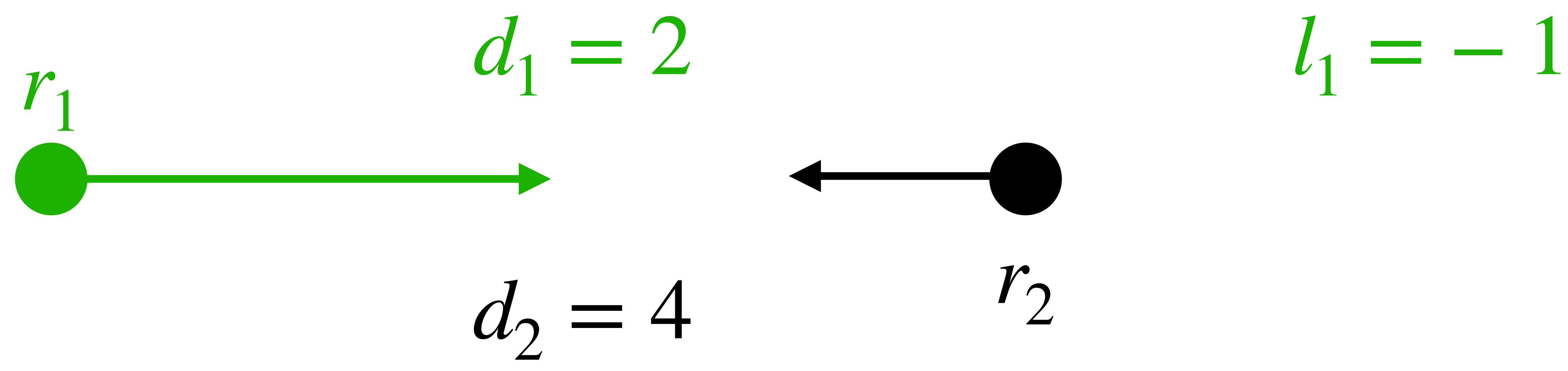

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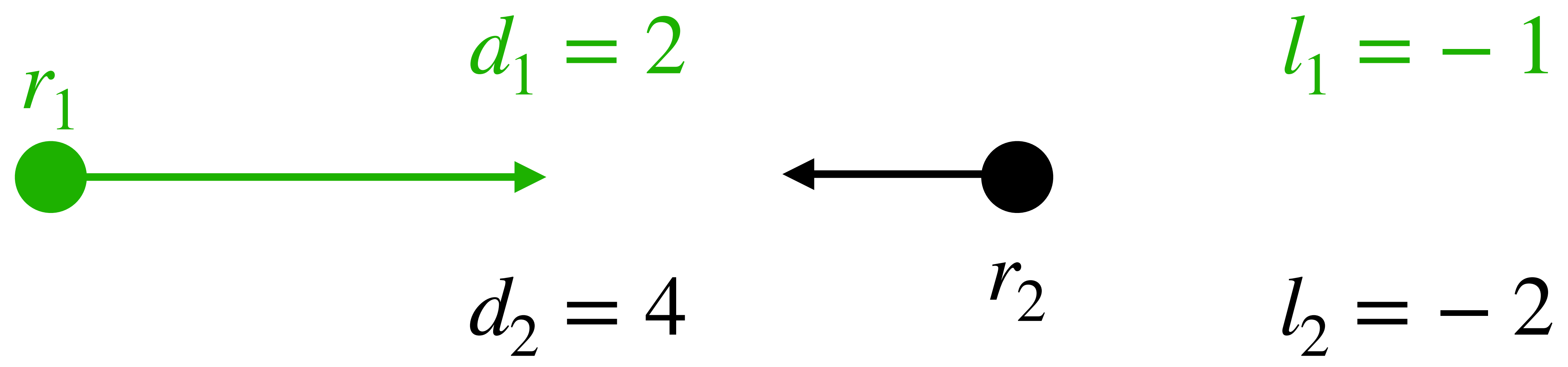

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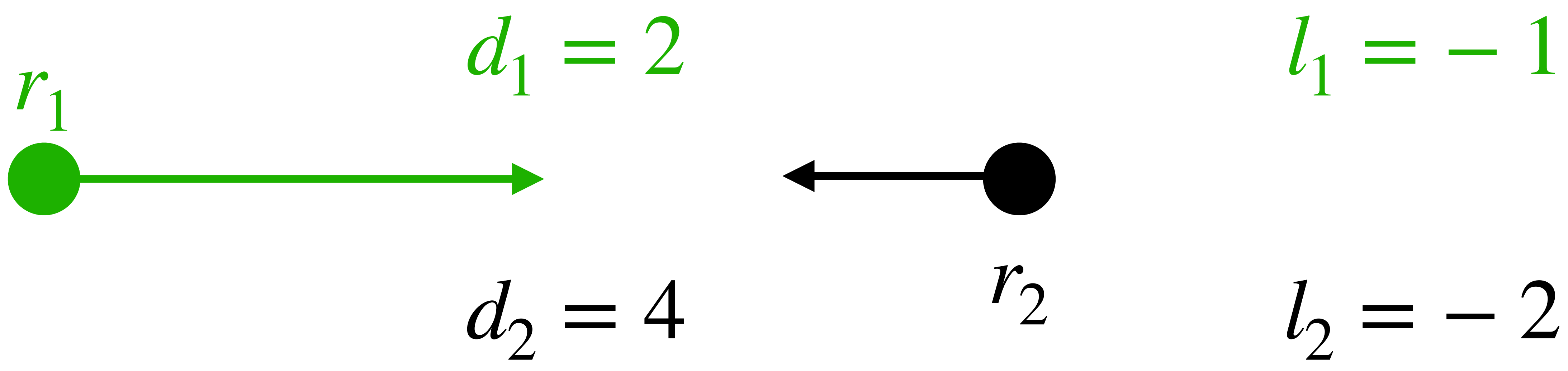

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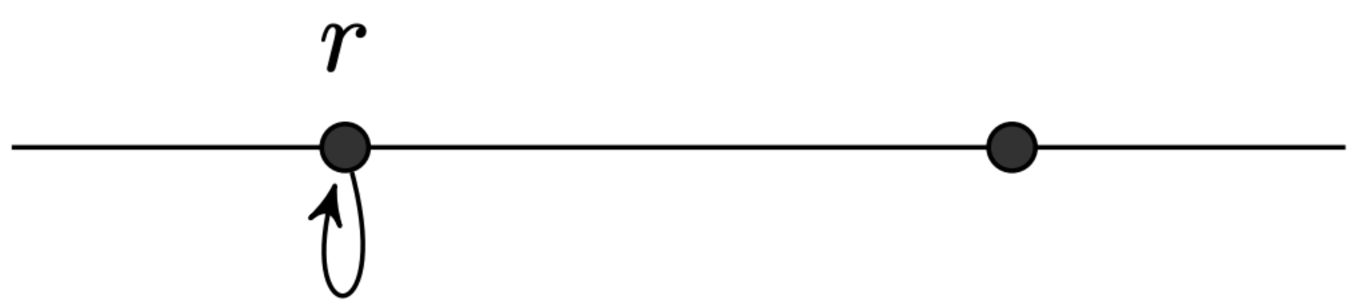

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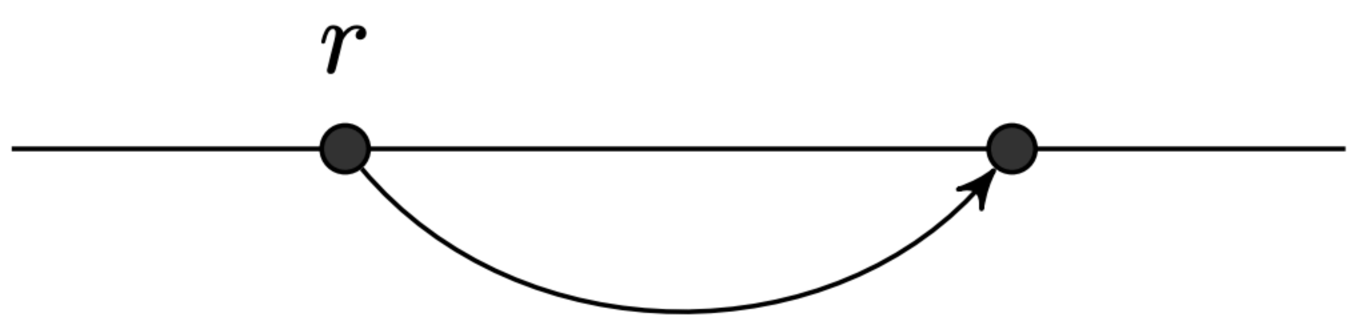

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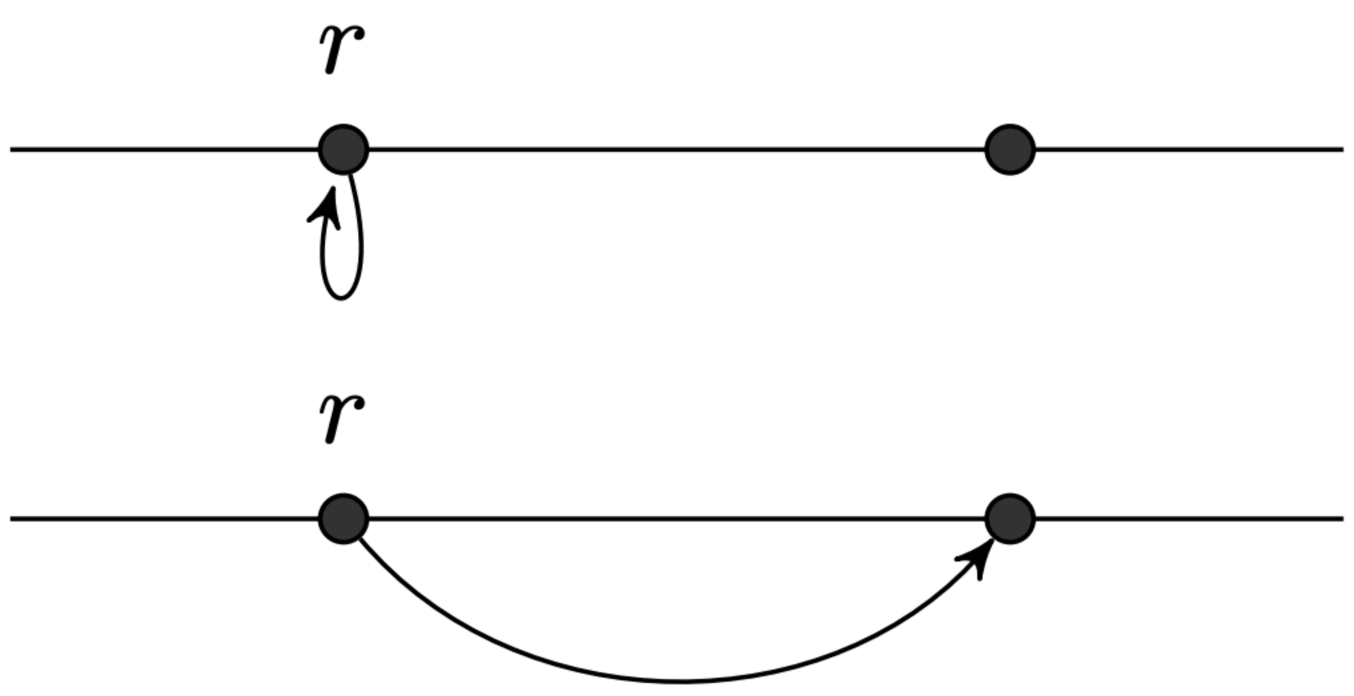

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Algo 1 solves rendezvous in SSYNC

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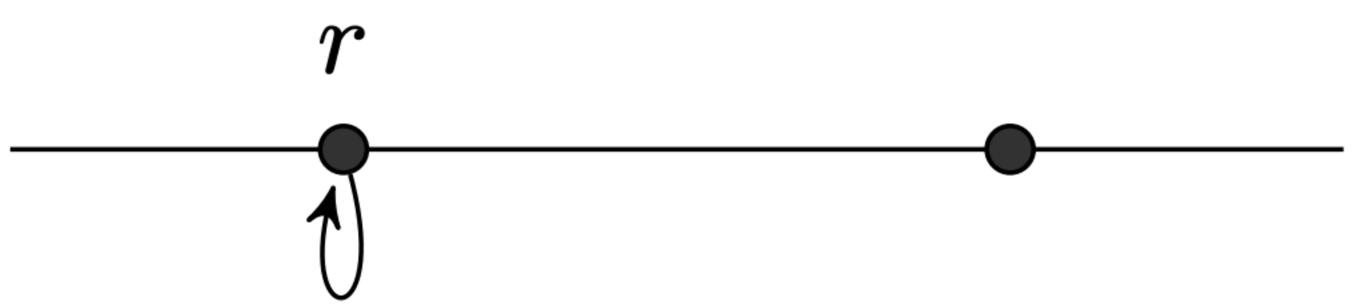

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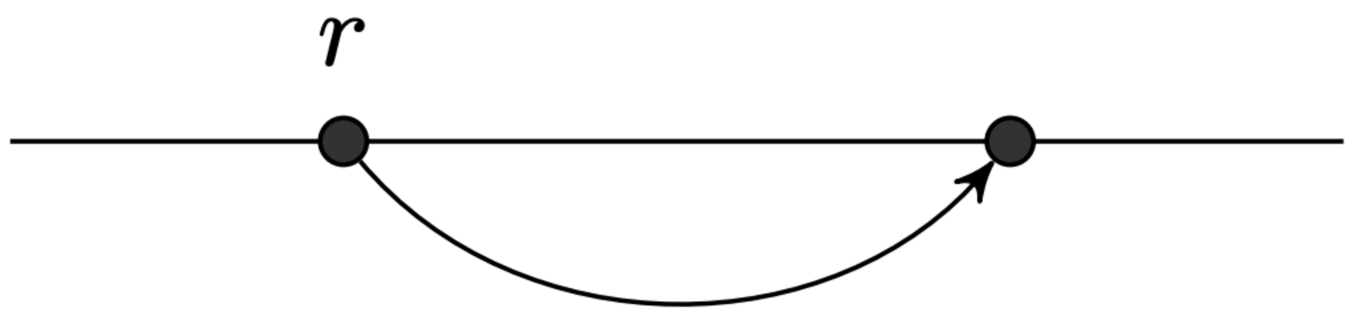

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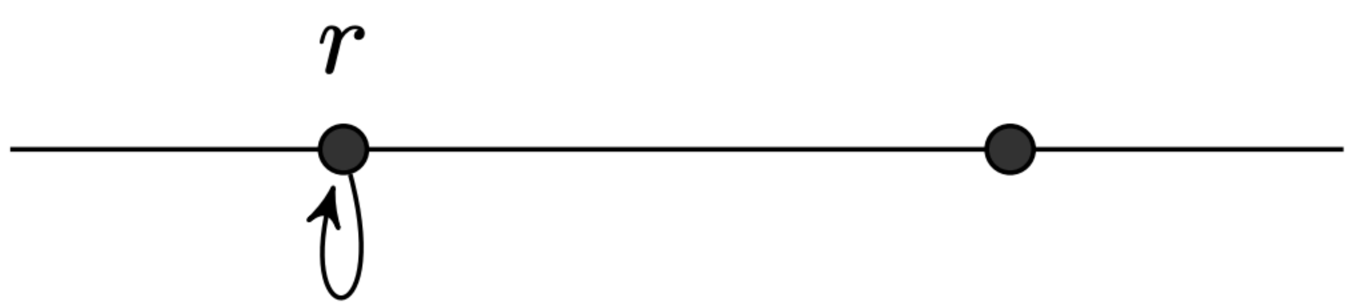

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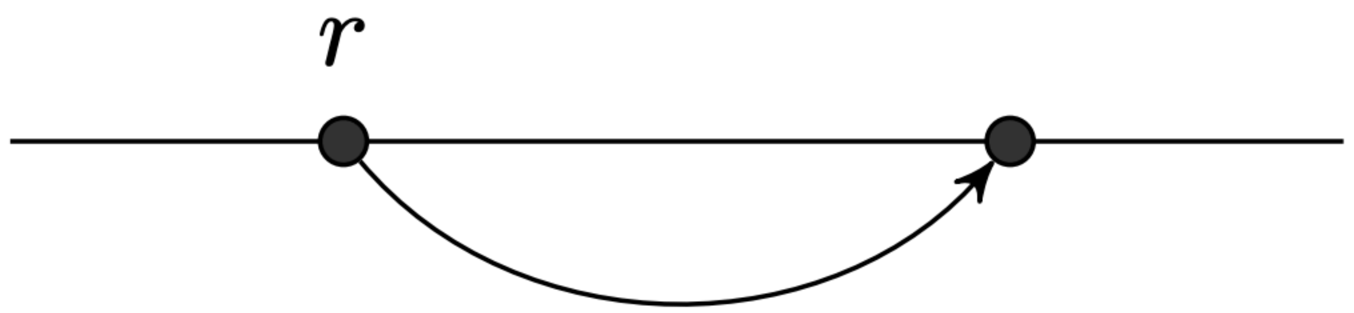

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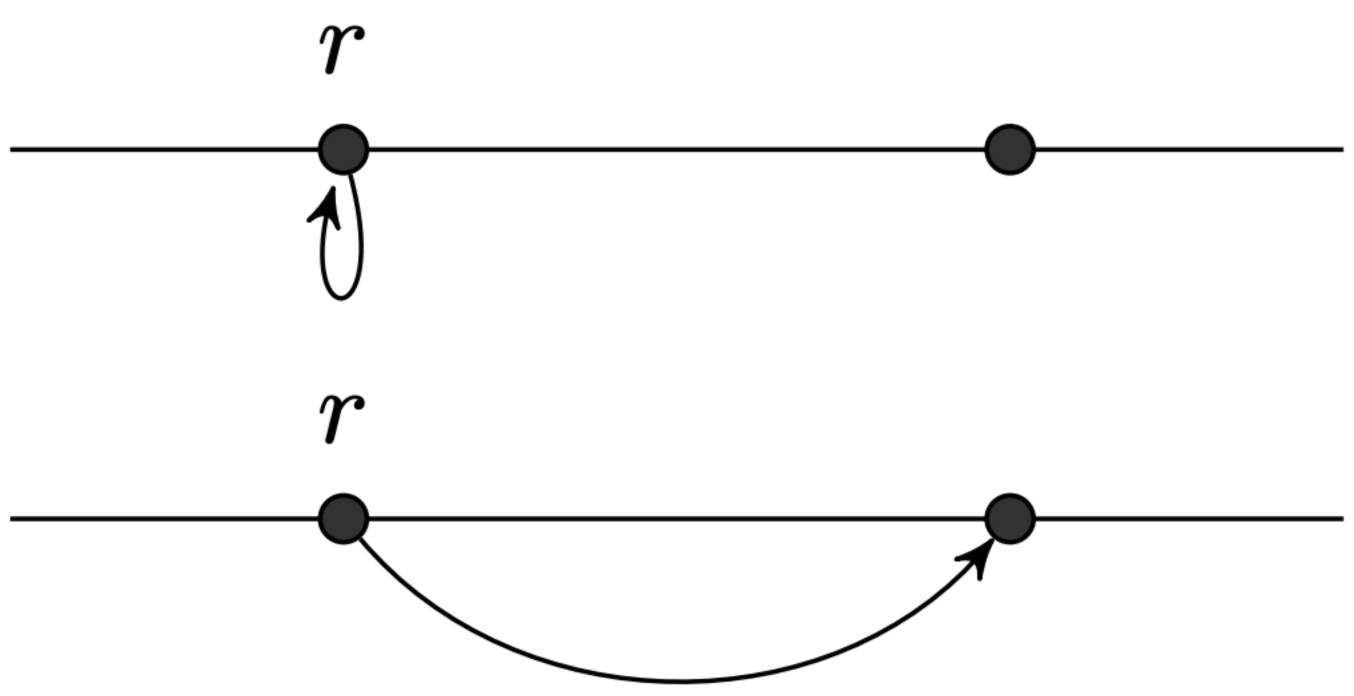

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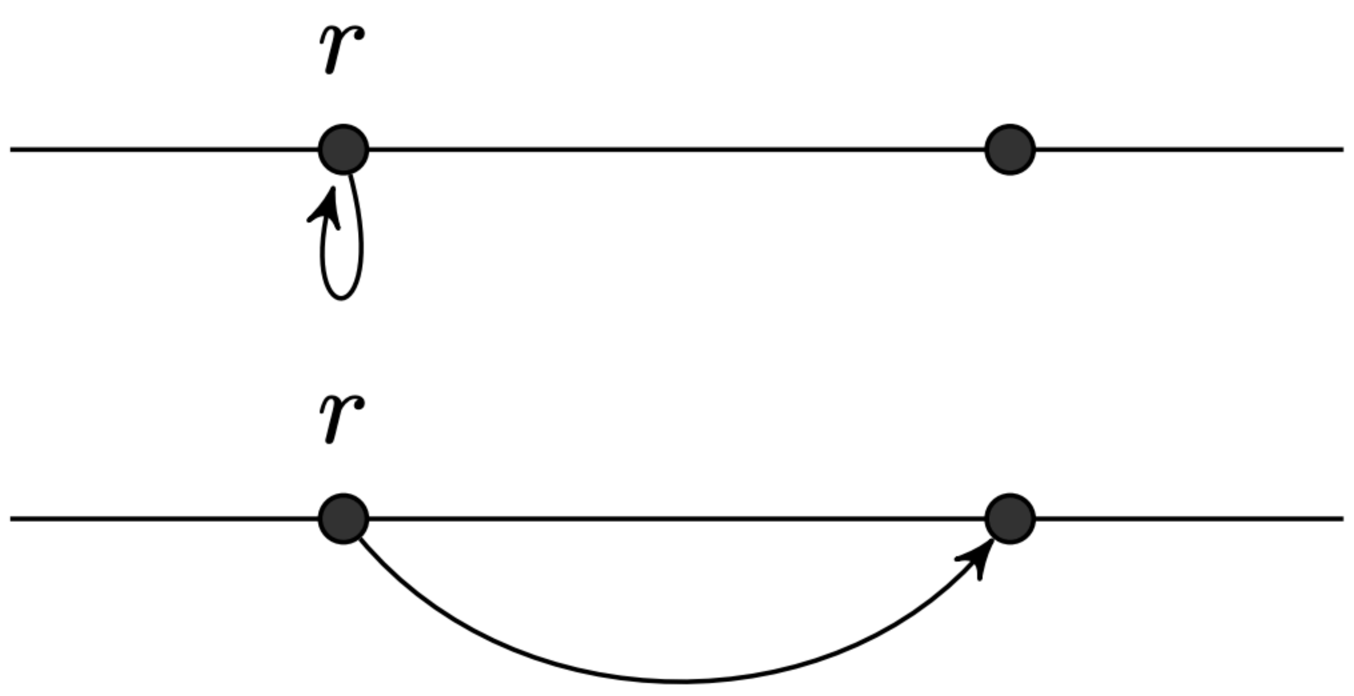

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Eventually, the distance between the robots is smaller than δ, hence robots eventually reach their target.

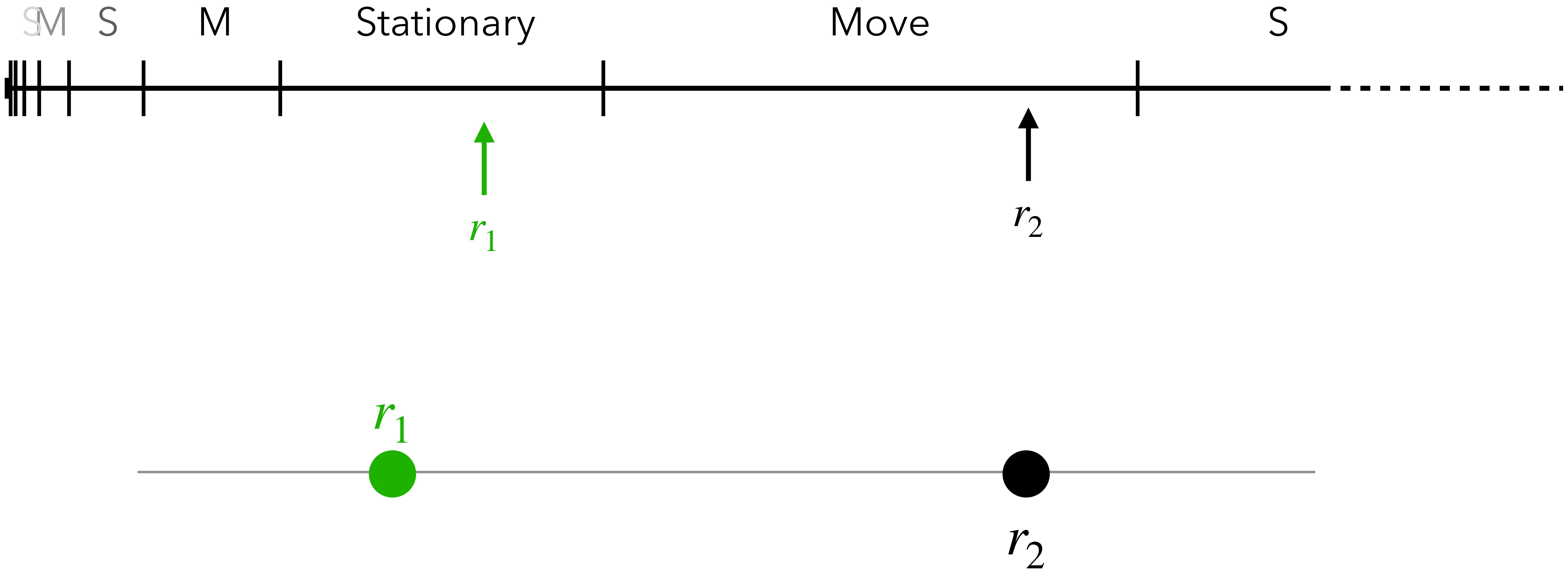
Rendezvous

$$\rho = [\rho_{min}, \rho_{\max}]$$

Rendezvous when $\rho = [\rho_{min}, \rho_{\max}]$

Understanding Levels

When $\rho = 2$



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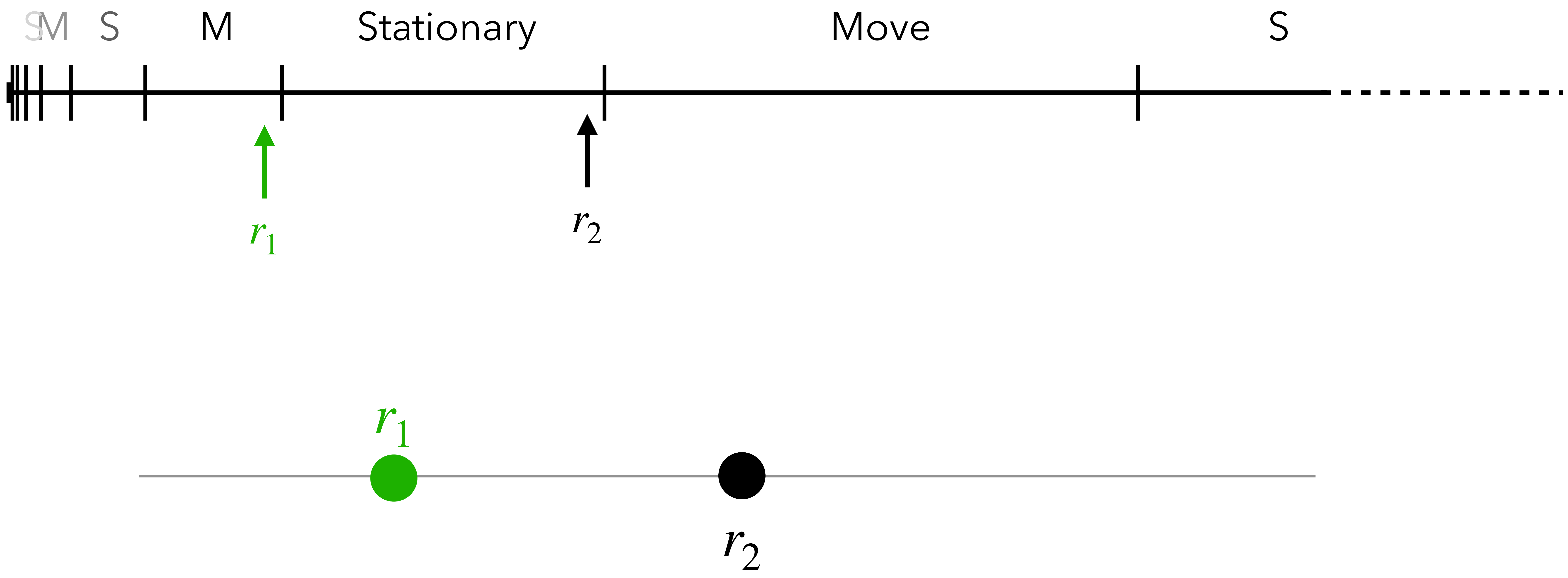

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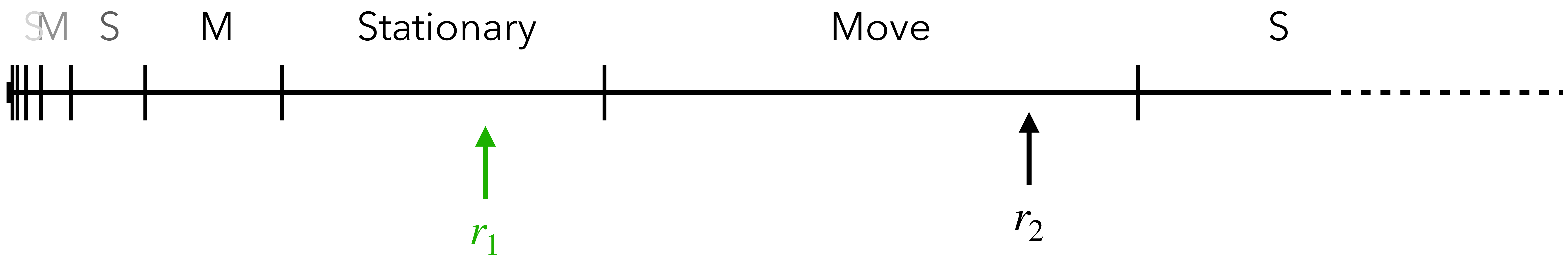

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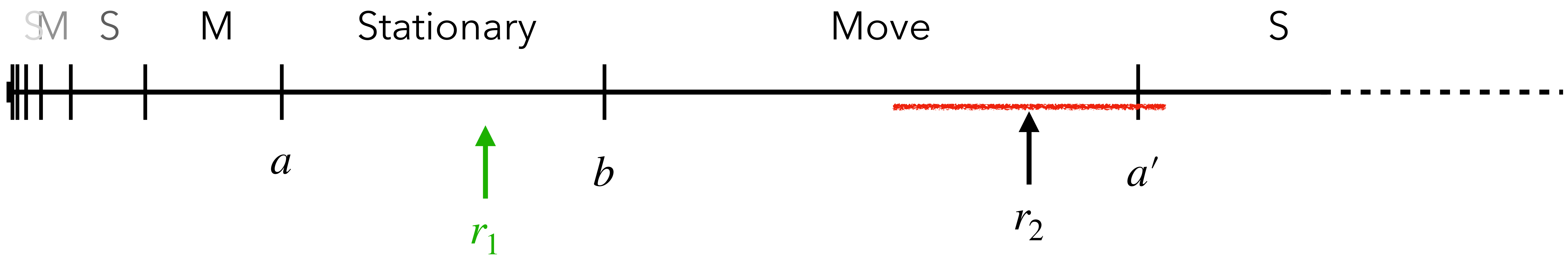

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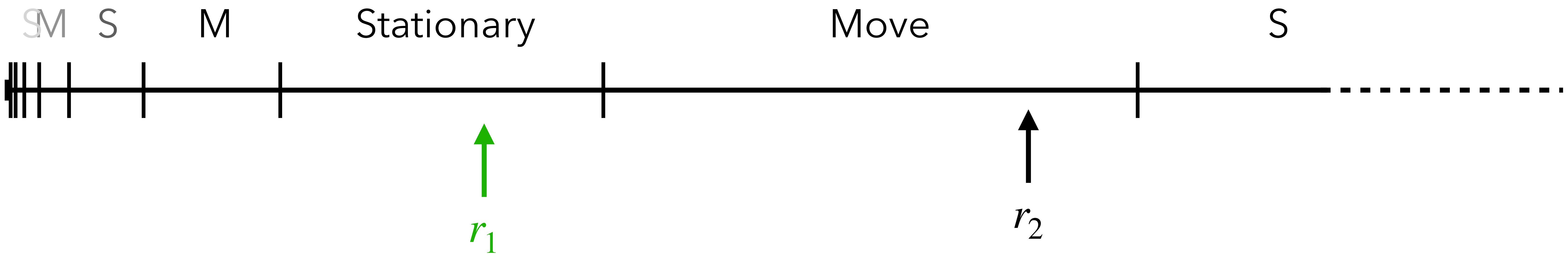

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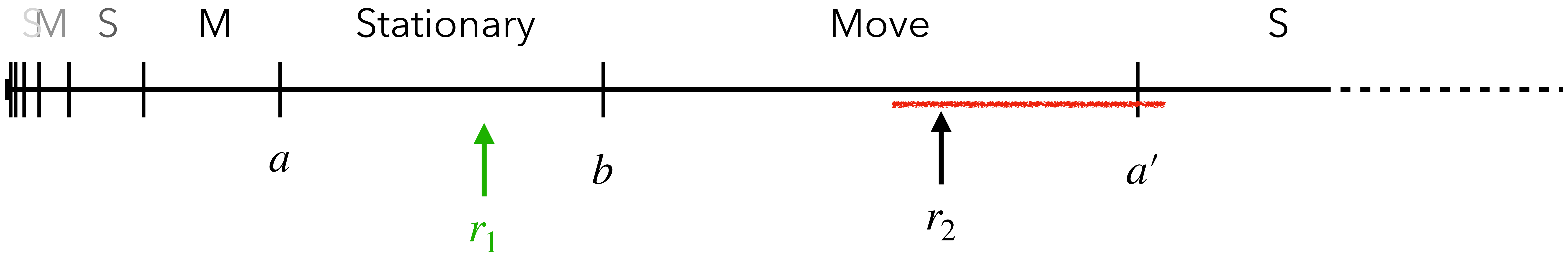

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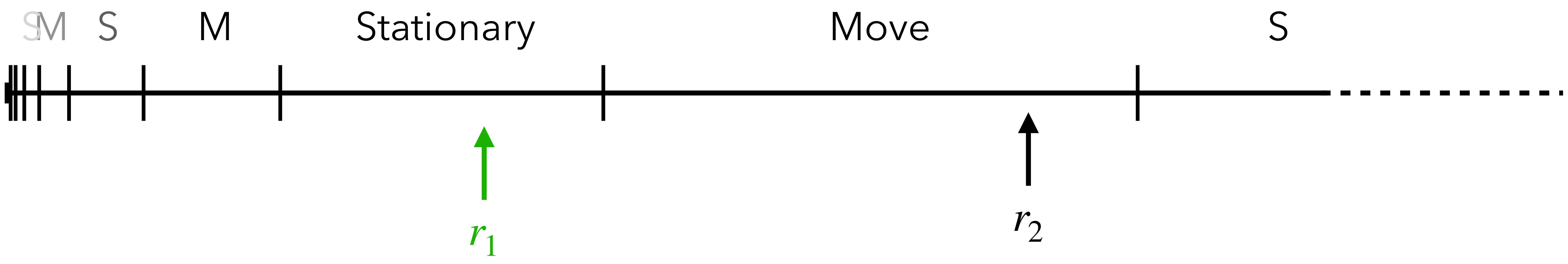

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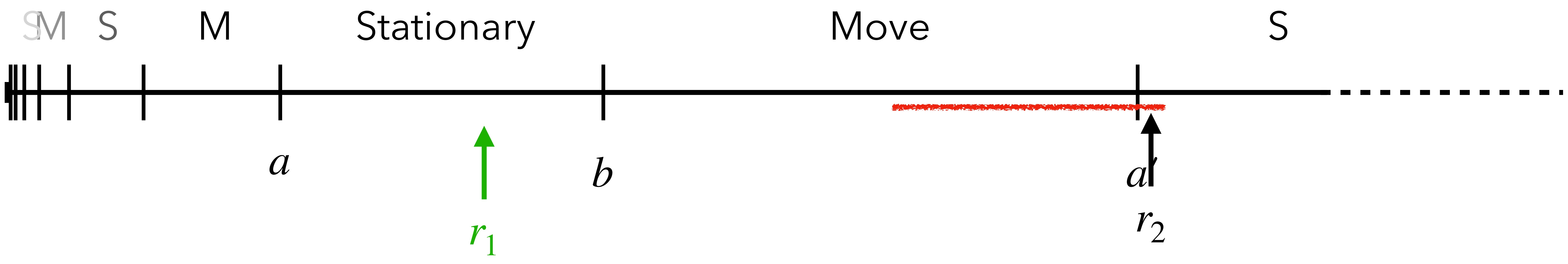

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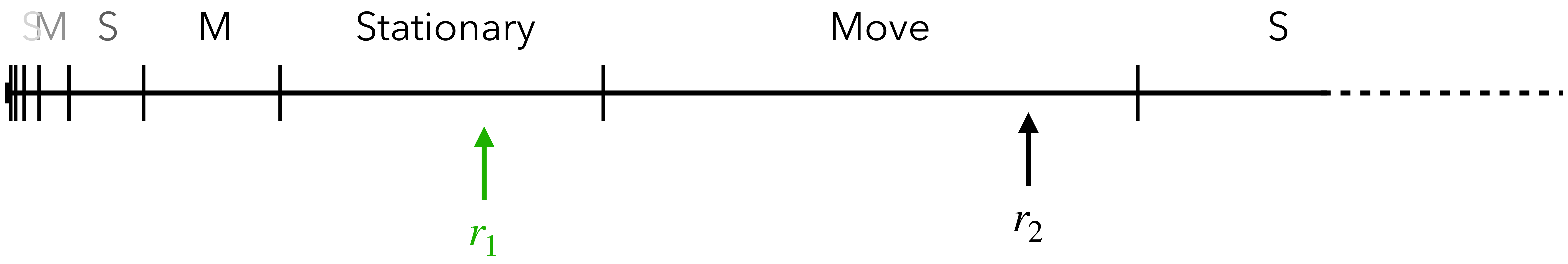

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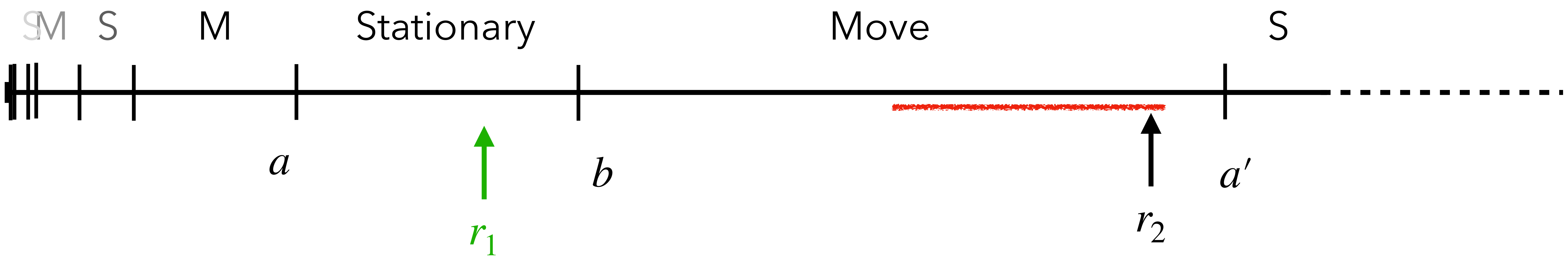

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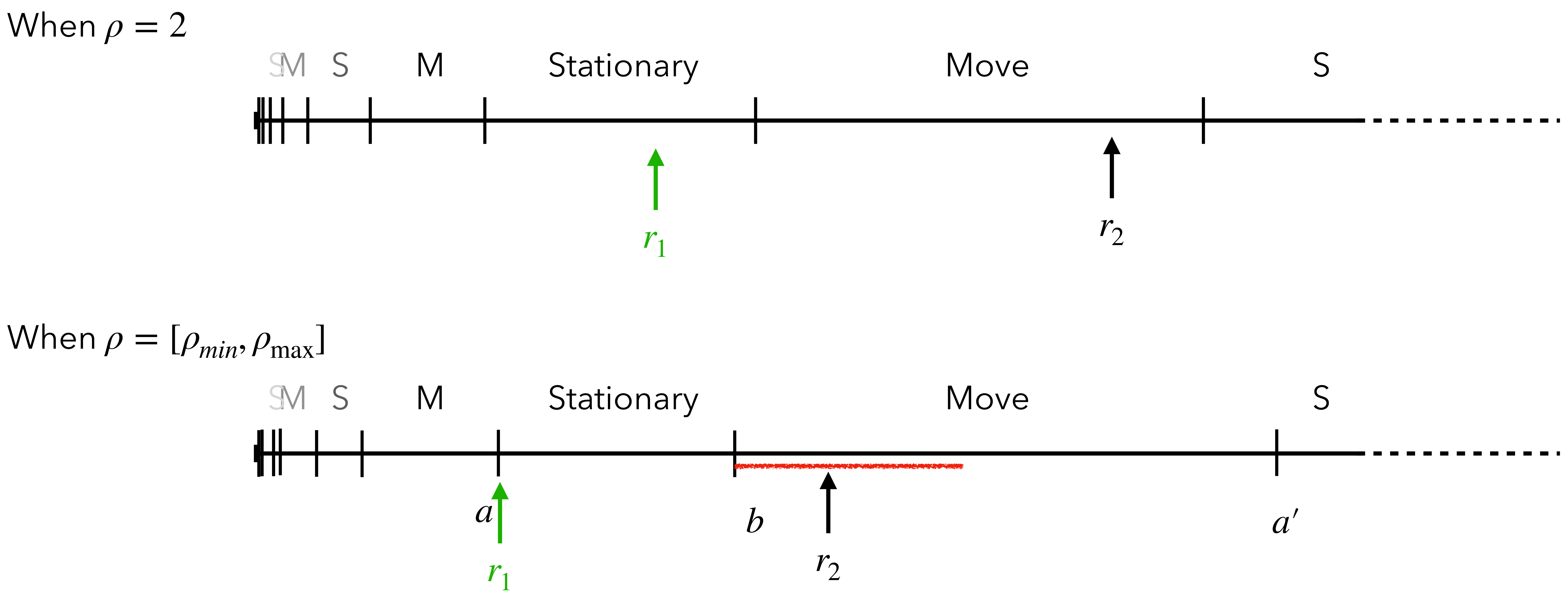

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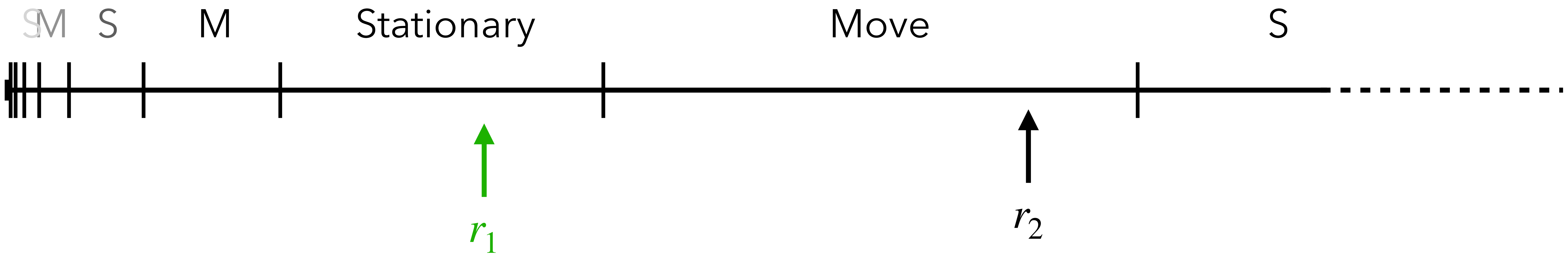

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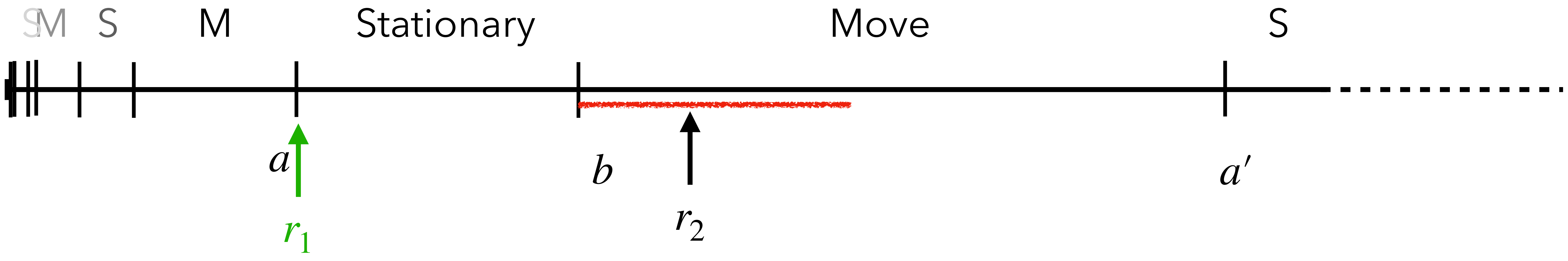

$$b = a \times \rho_{\min}$$

Rendezvous when $\rho = [\rho_{min}, \rho_{\max}]$

Understanding Levels

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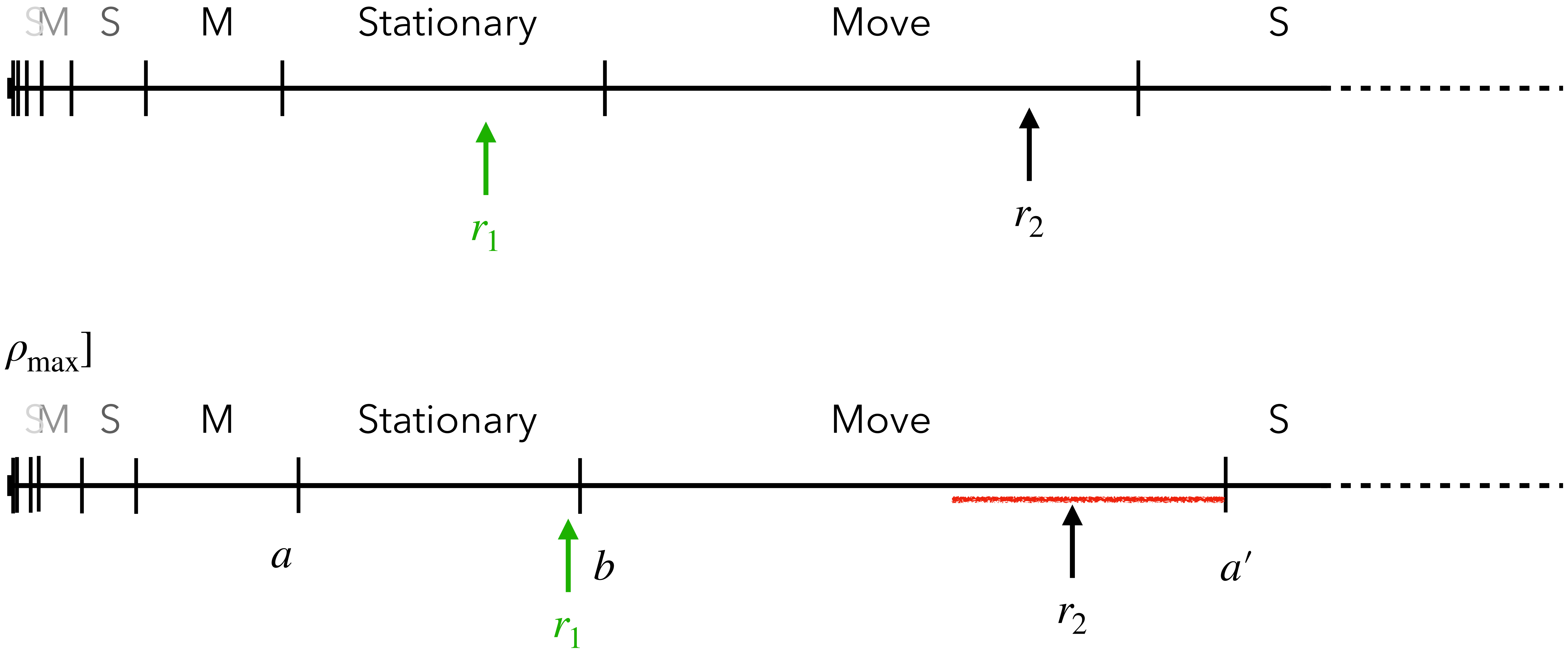

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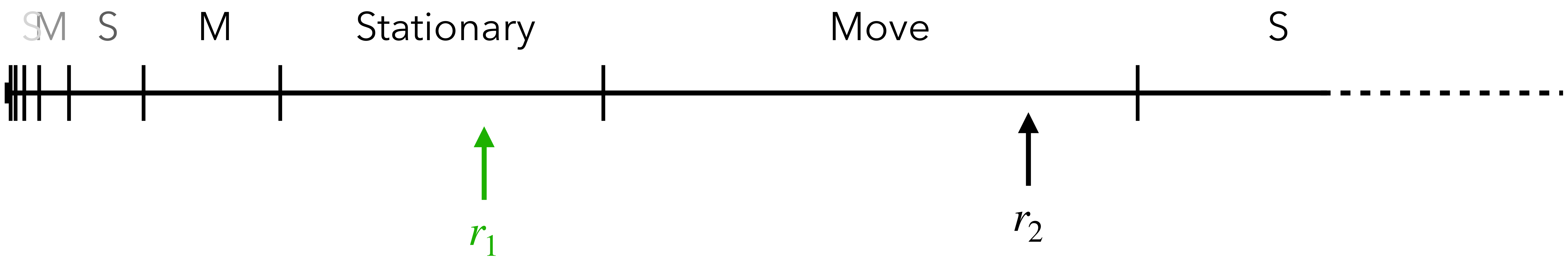

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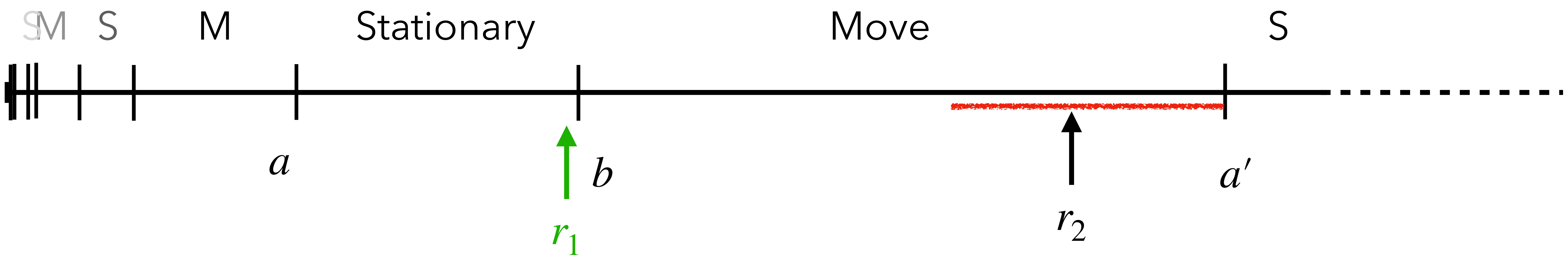


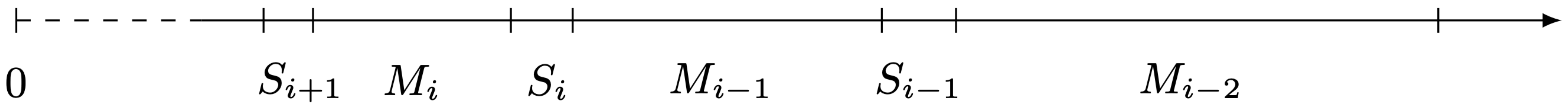
$b = a \times \rho_{\min}$ $a' = b \times \rho_{\max}$

Rendezvous when $\rho = [\rho_{min}, \rho_{\max}]$

Robots Levels

$$\forall i \in \mathbb{Z} \qquad S_i = [\rho_{\min}^{-i}\rho_{\max}^{-i}, \rho_{\min}^{-(i-1)}\rho_{\max}^{-i})$$

$$M_i = [\rho_{\min}^{-i}\rho_{\max}^{-(i+1)}, \rho_{\min}^{-i}\rho_{\max}^{-i})$$



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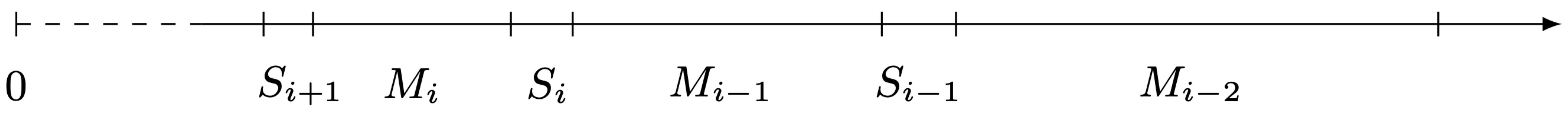

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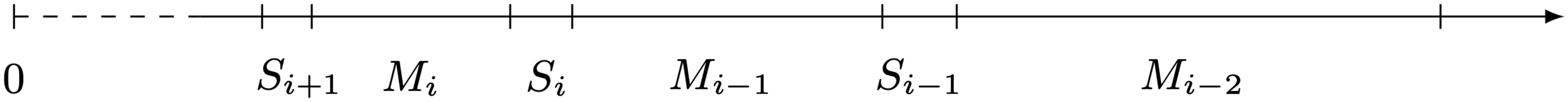

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Algo2:

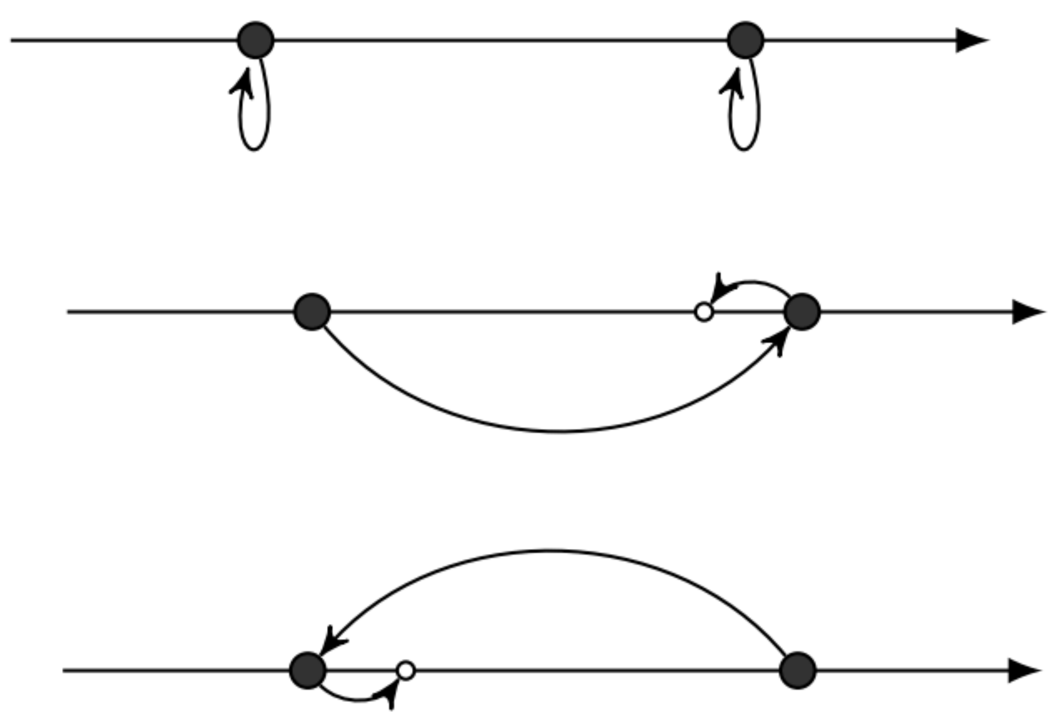
$d \in \mathcal{S}_0 \cup \mathcal{S}_1$

$d \in \mathcal{M}_0$

The right robot moves a distance $\mathfrak{s}(d)$

$d \in M_1$

The left robot moves a distance $\mathfrak{s}(d)$



With $\mathfrak{s}(d) \in \mathcal{S}_0$

Rendezvous when $\rho = [\rho_{min}, \rho_{\max}]$

Algo2 execution

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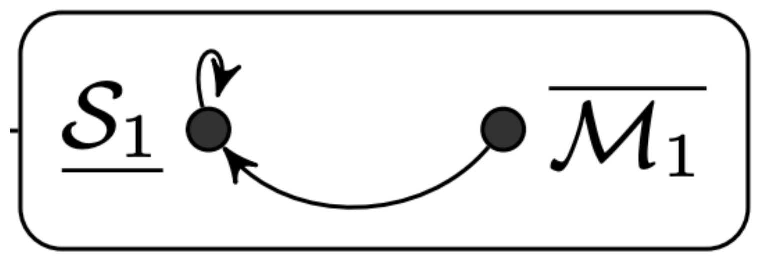

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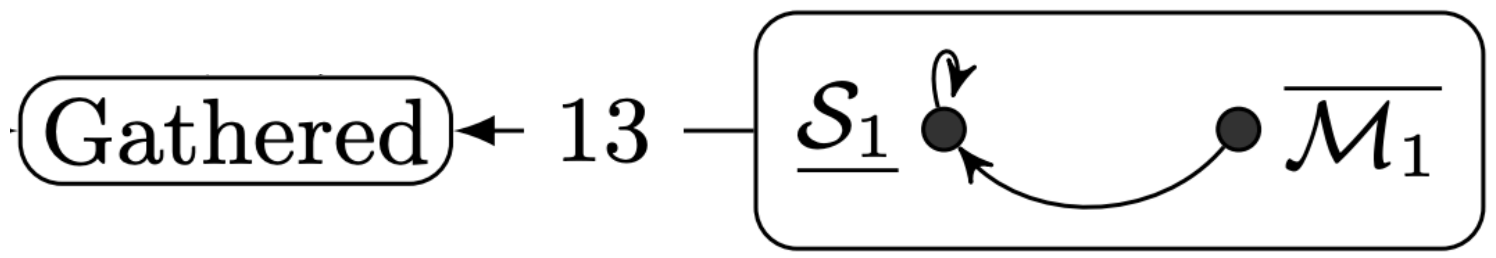

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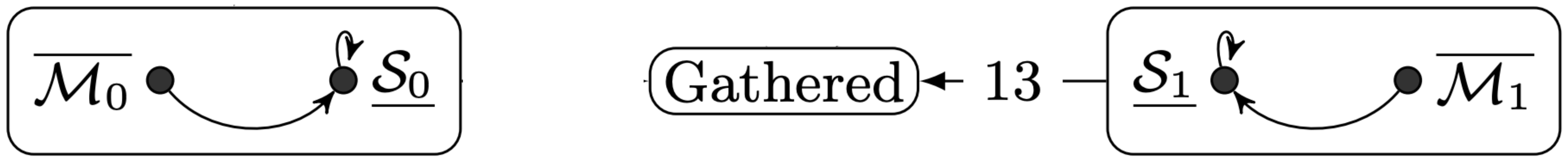

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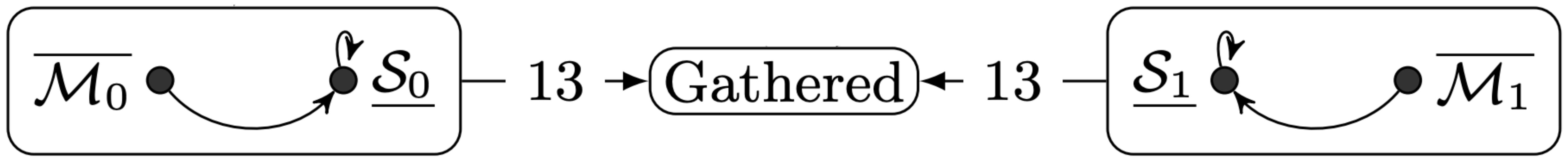

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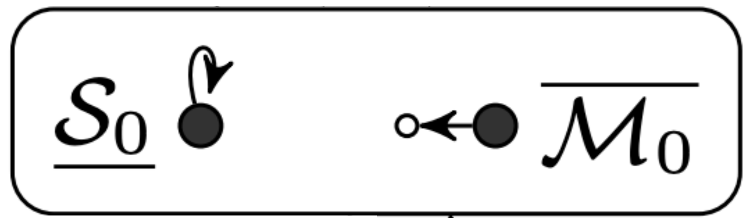

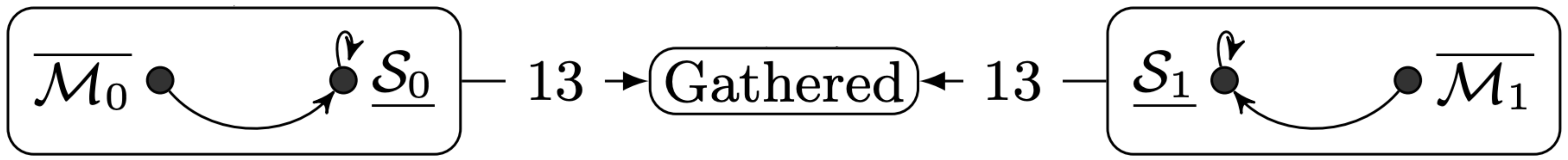

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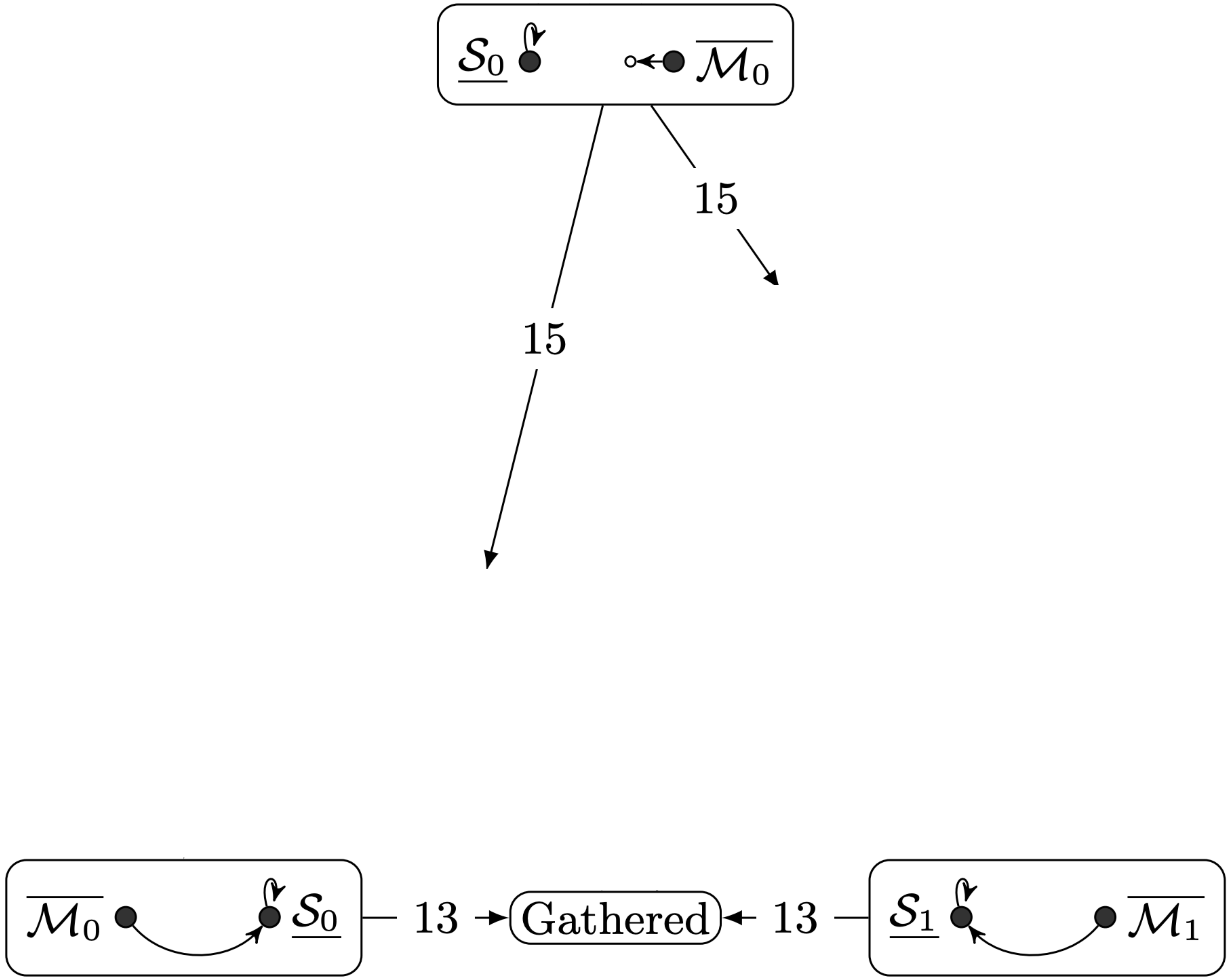

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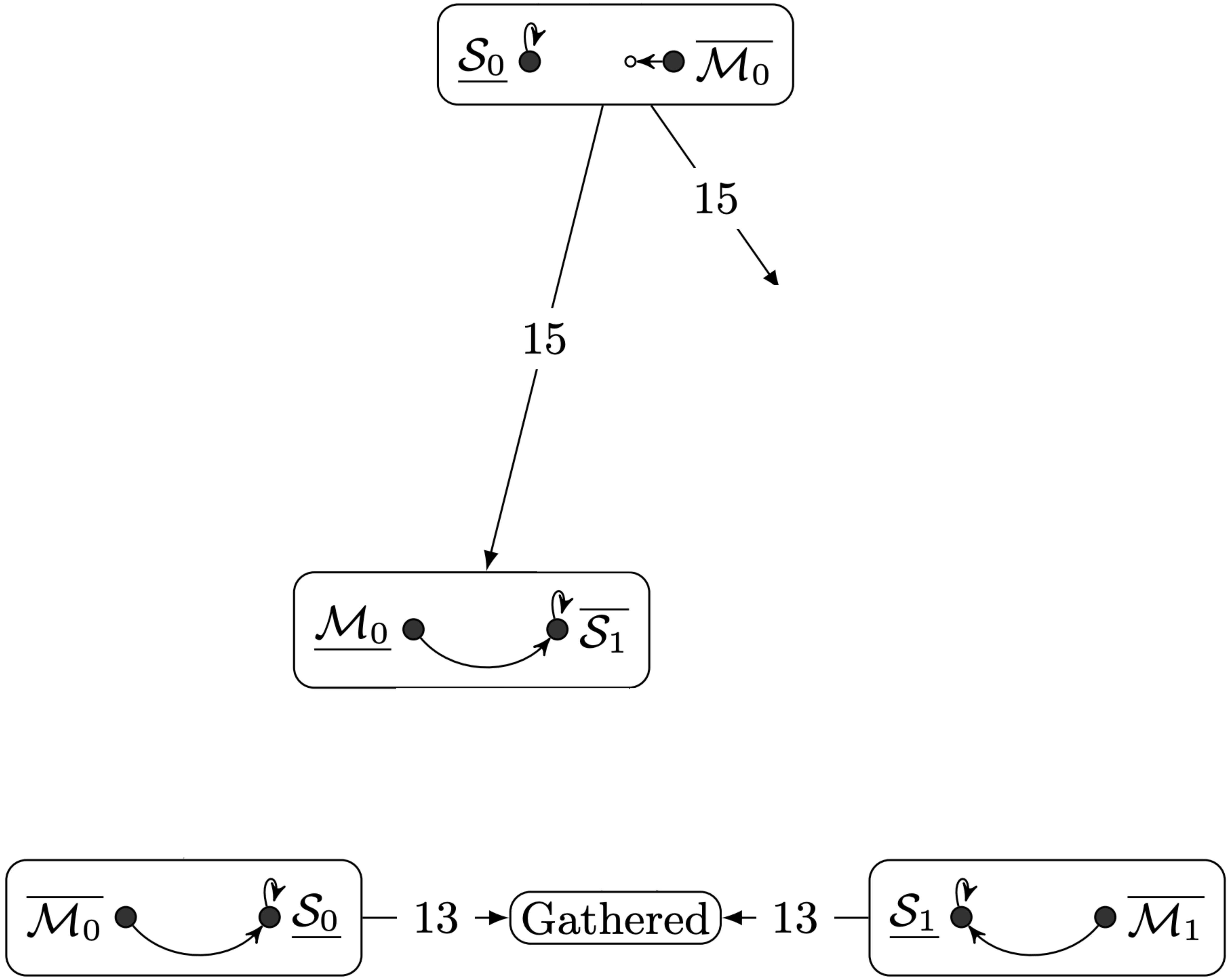

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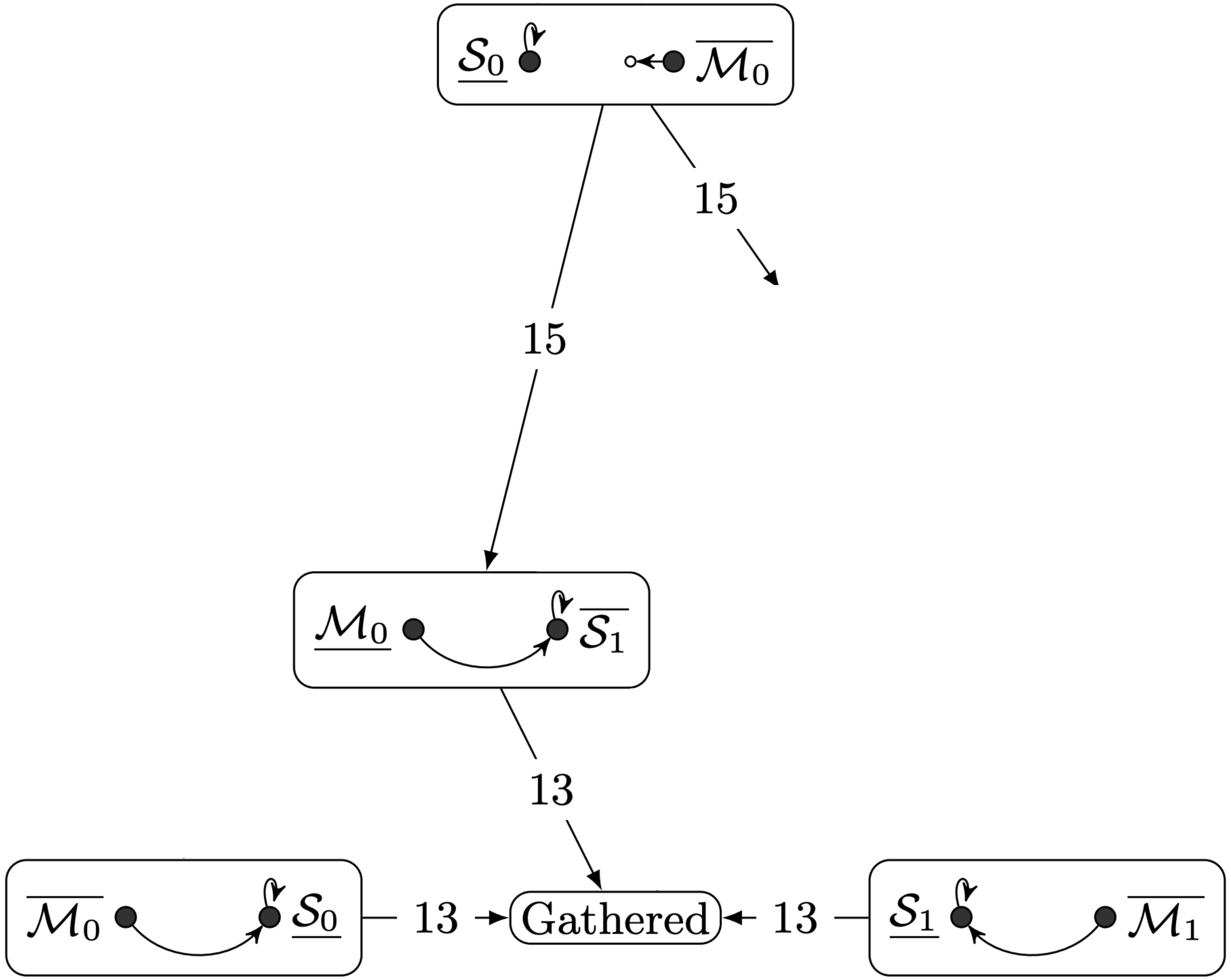

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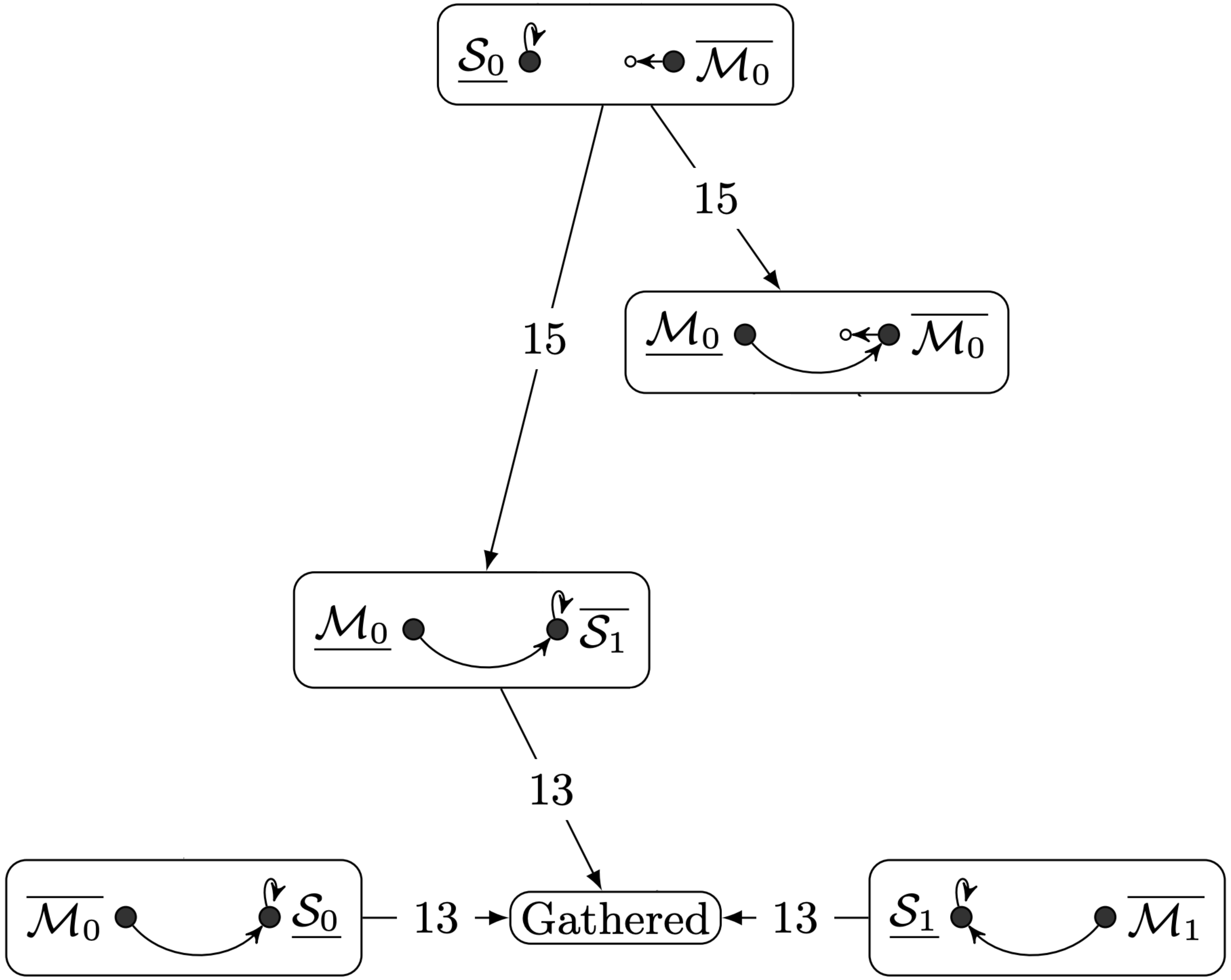

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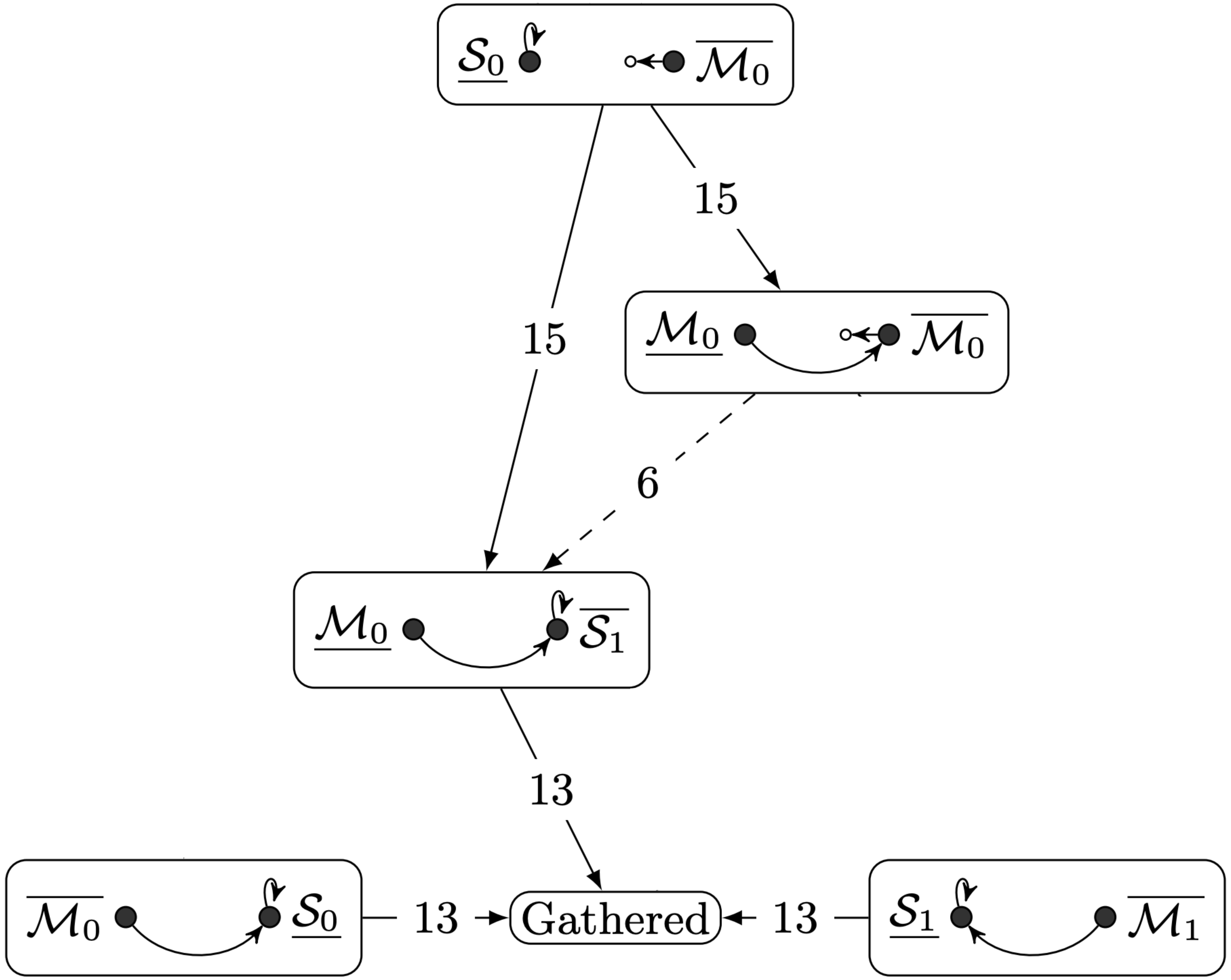

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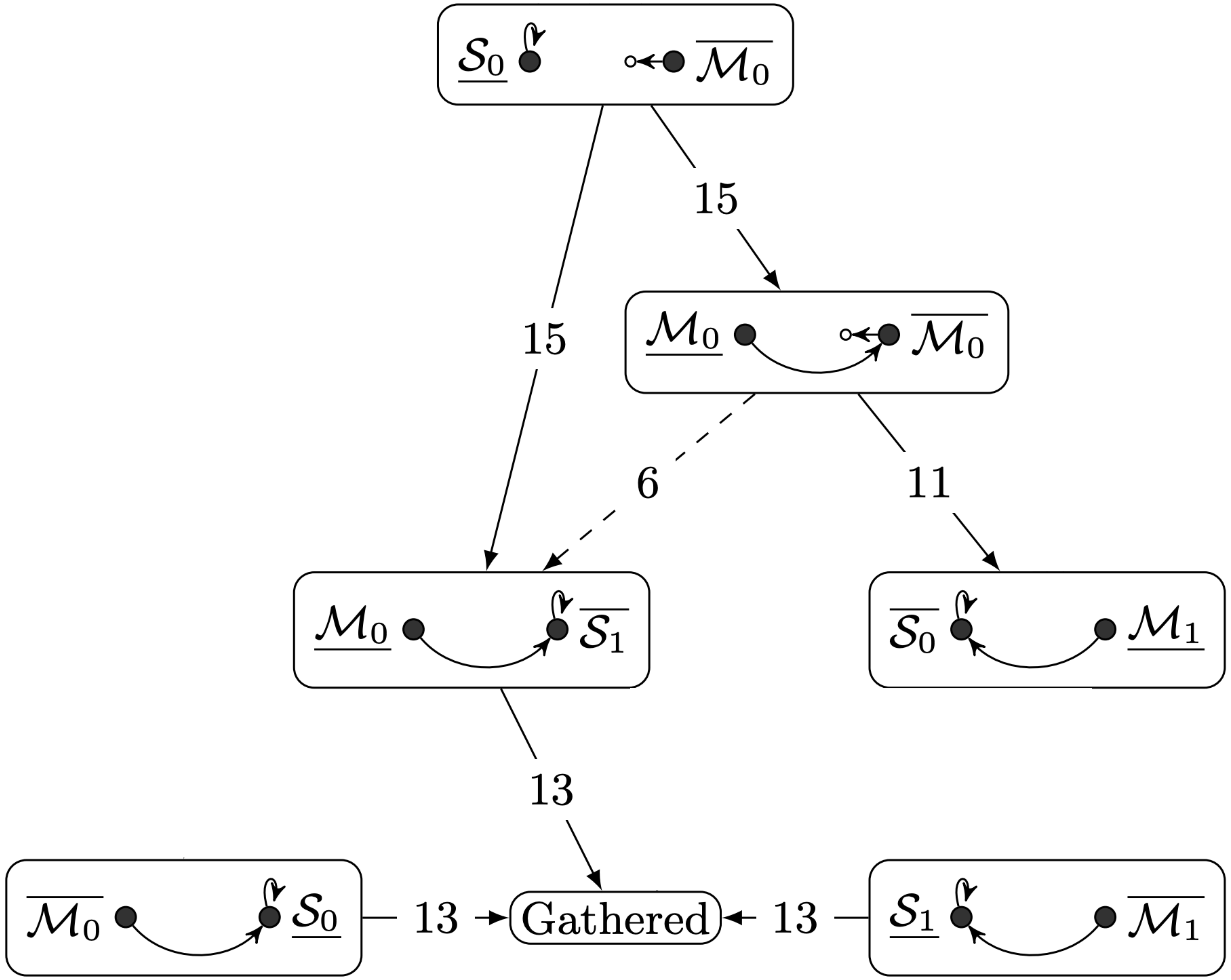

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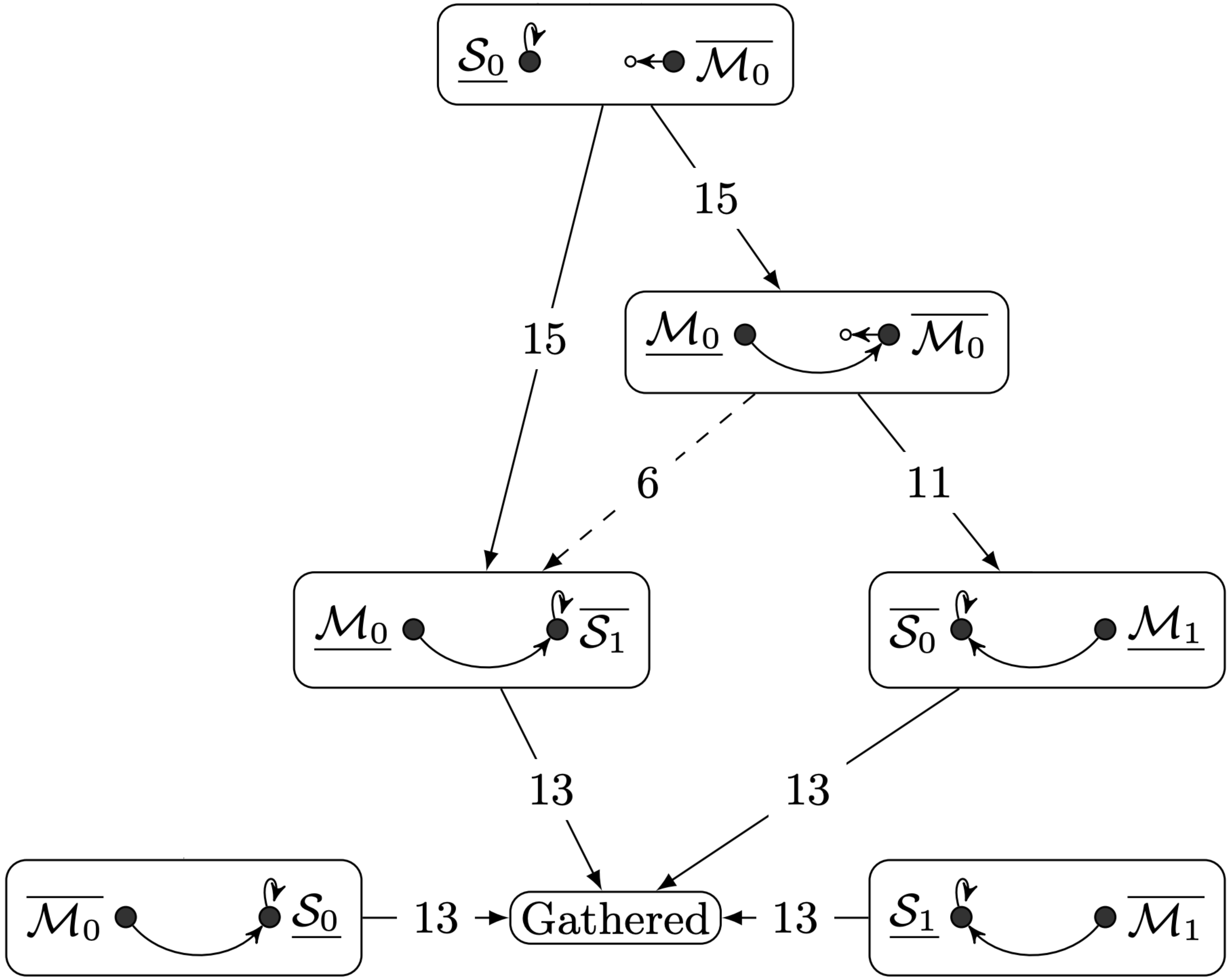

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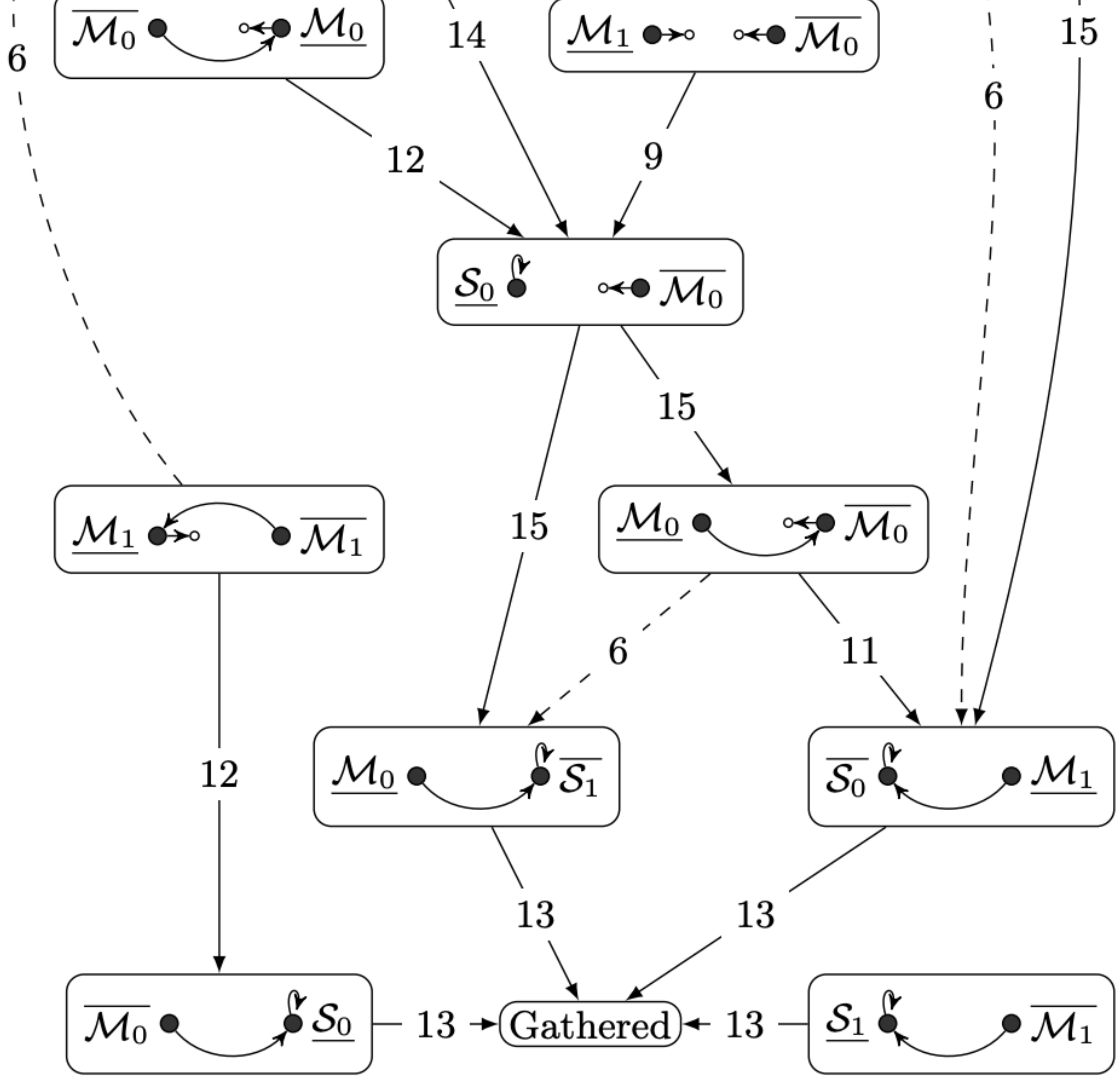

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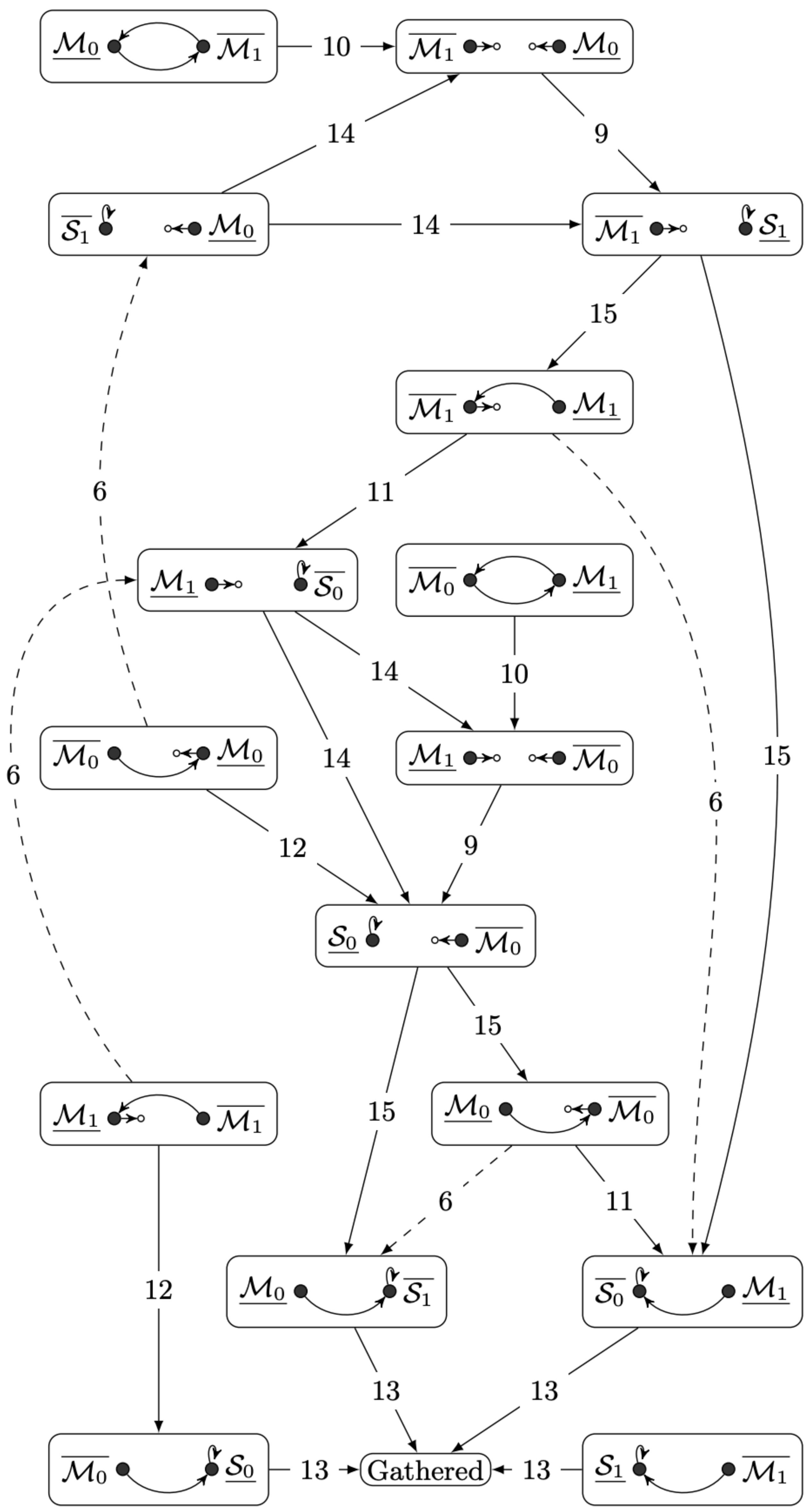

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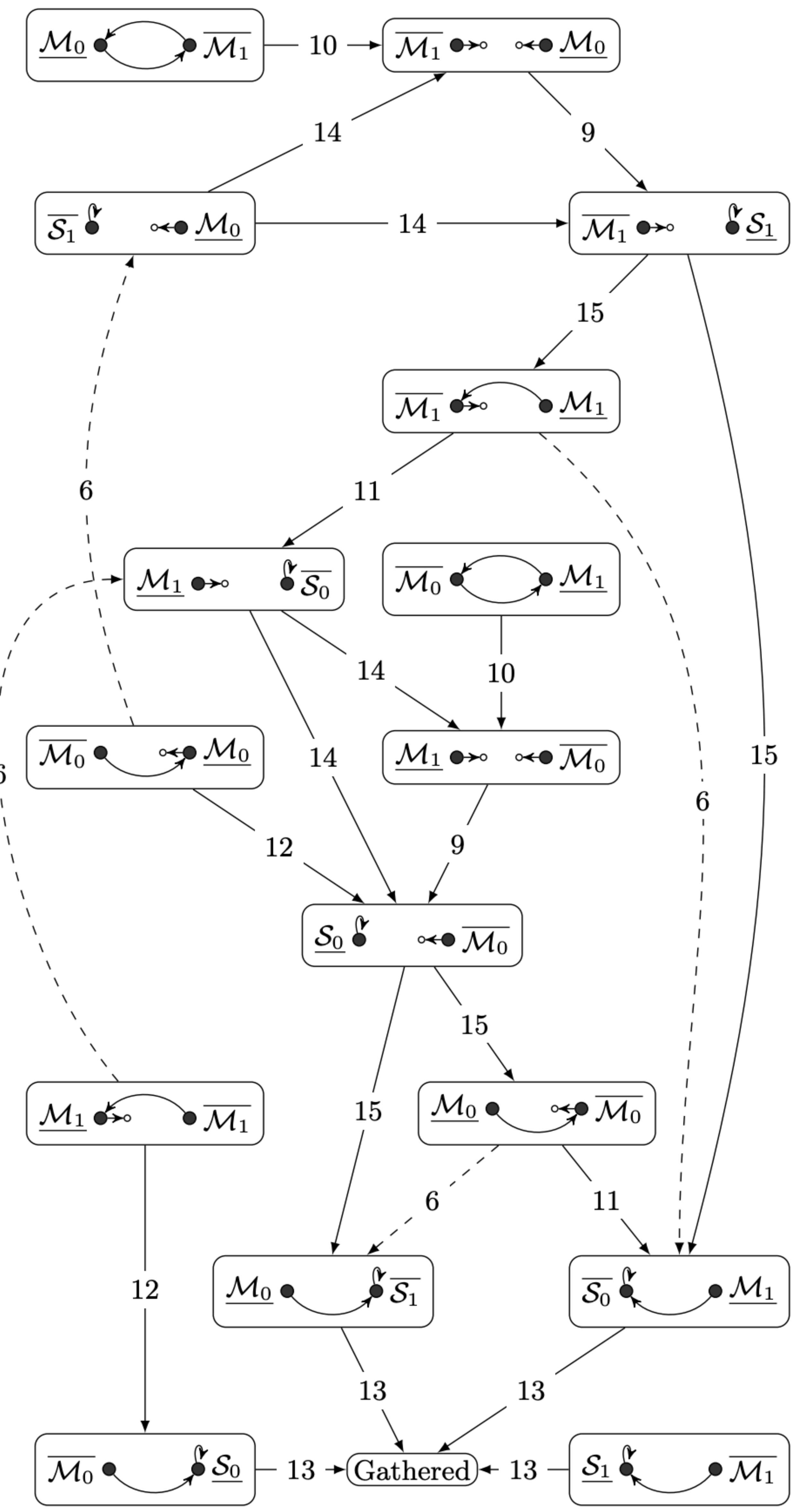

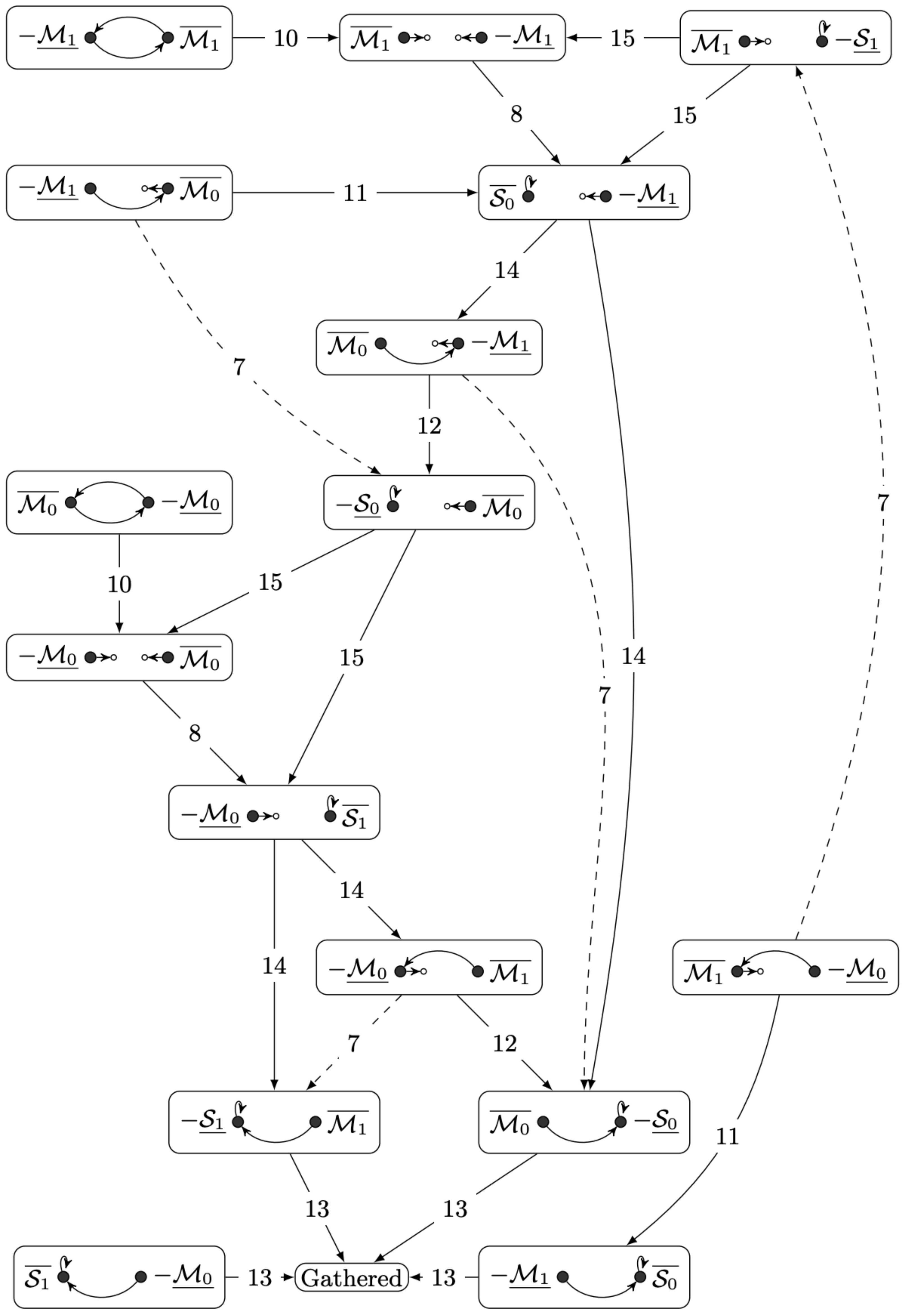

Conclusion

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Thank you!