Online Space-time Travel Planning in Dynamic Graphs

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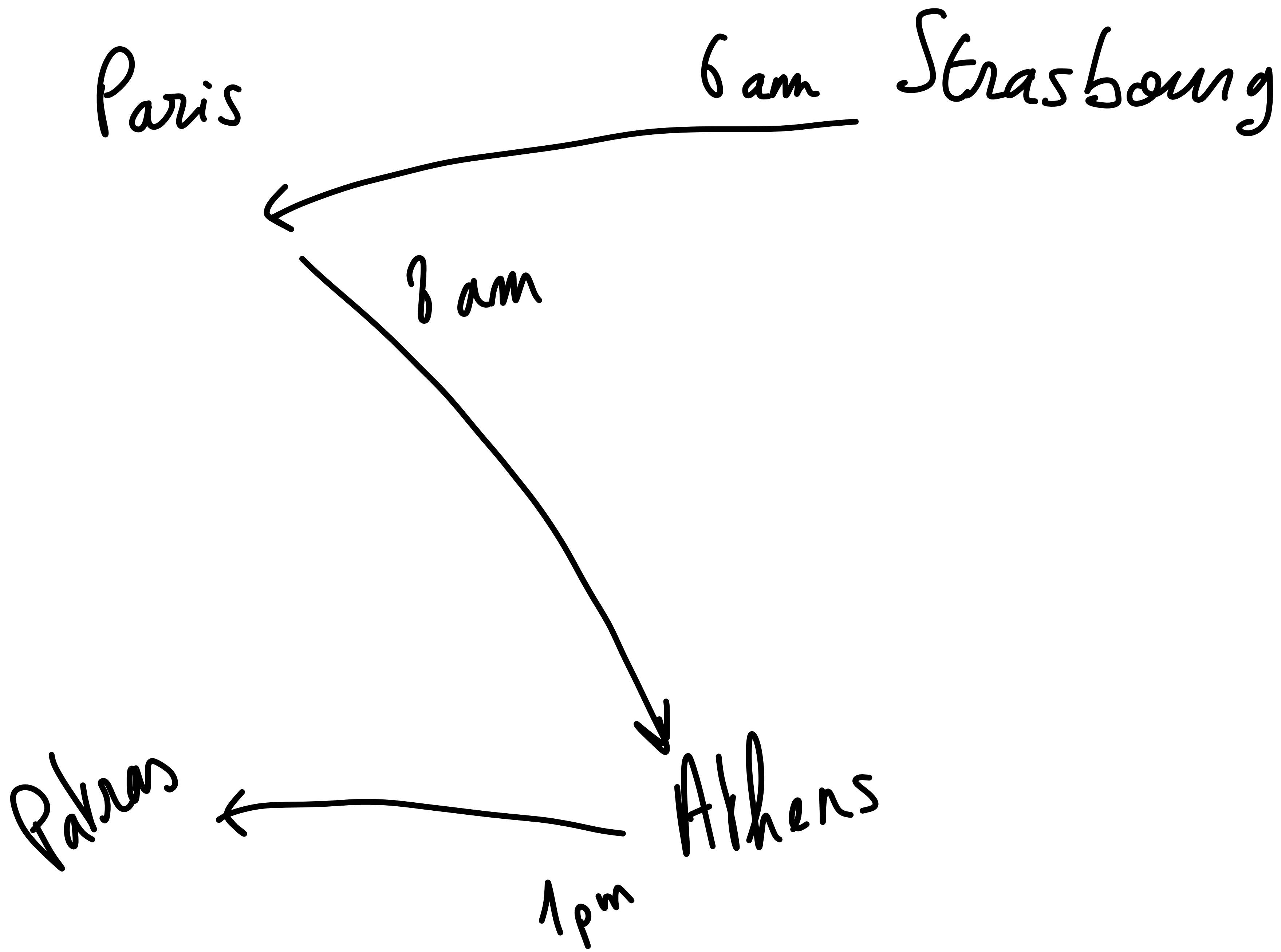
Université de Strasbourg

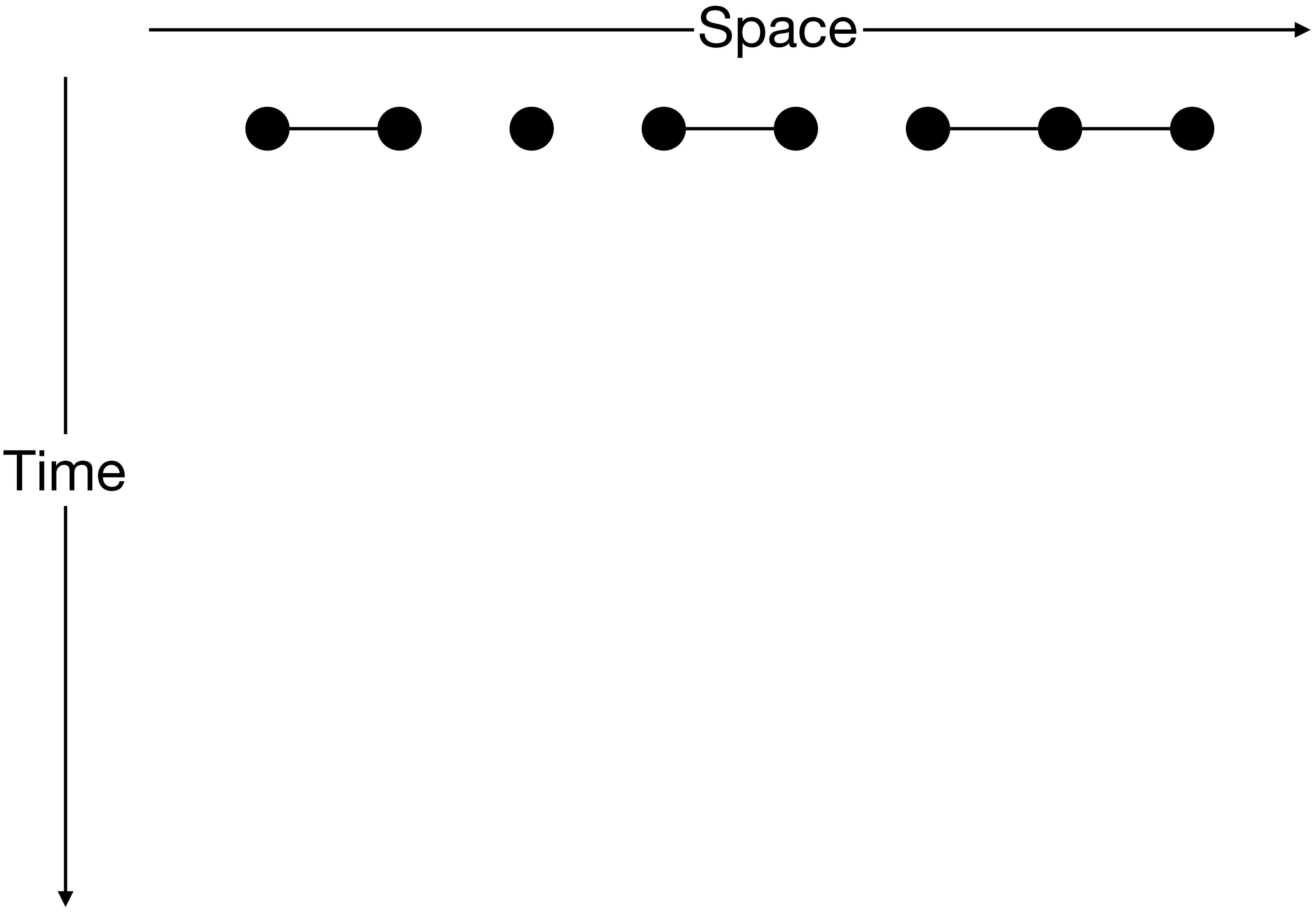
SORBONNE UNIVERSITÉ

SAND 2024

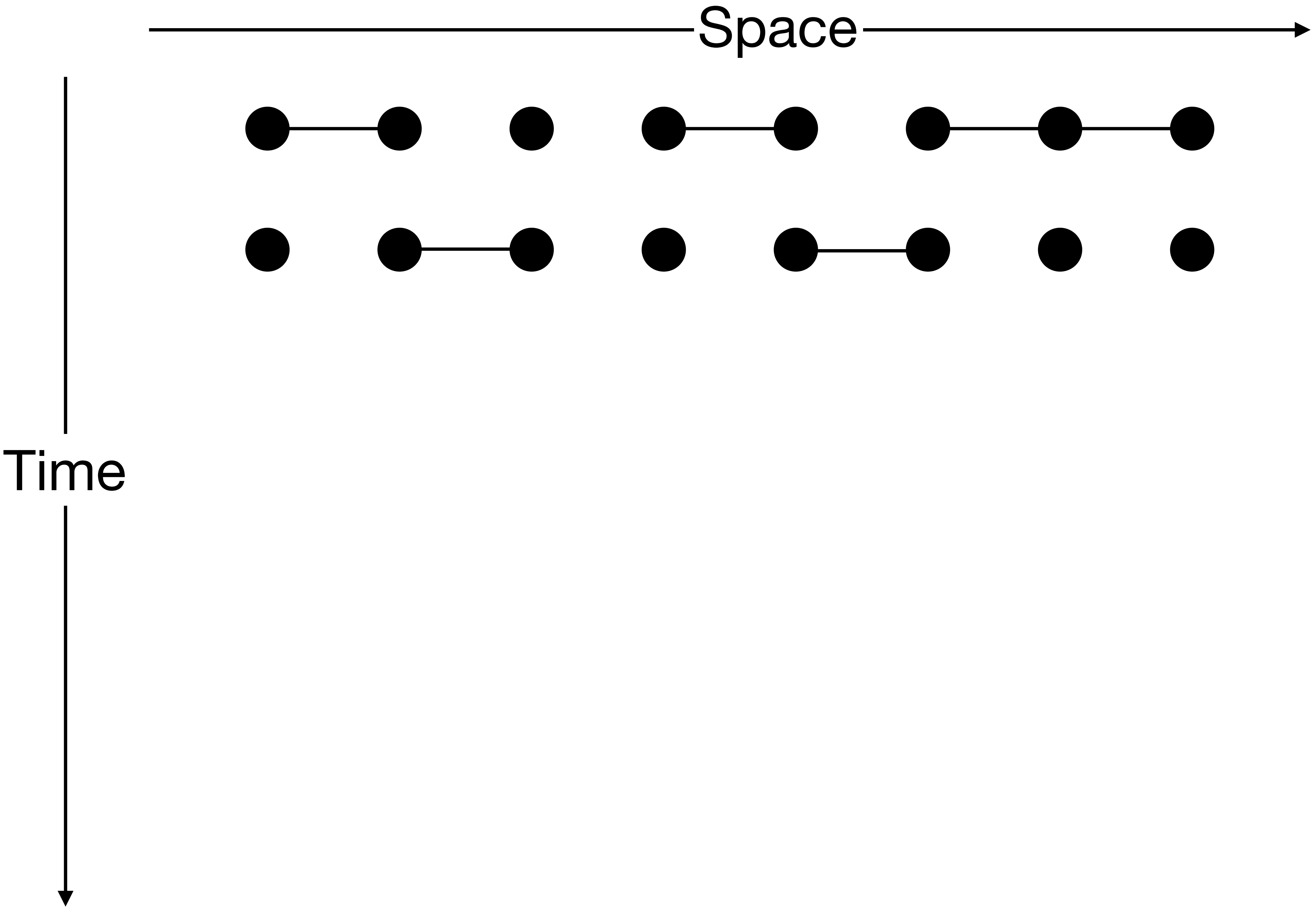
Dynamic Graphs



Evolving Graphs



Evolving Graphs



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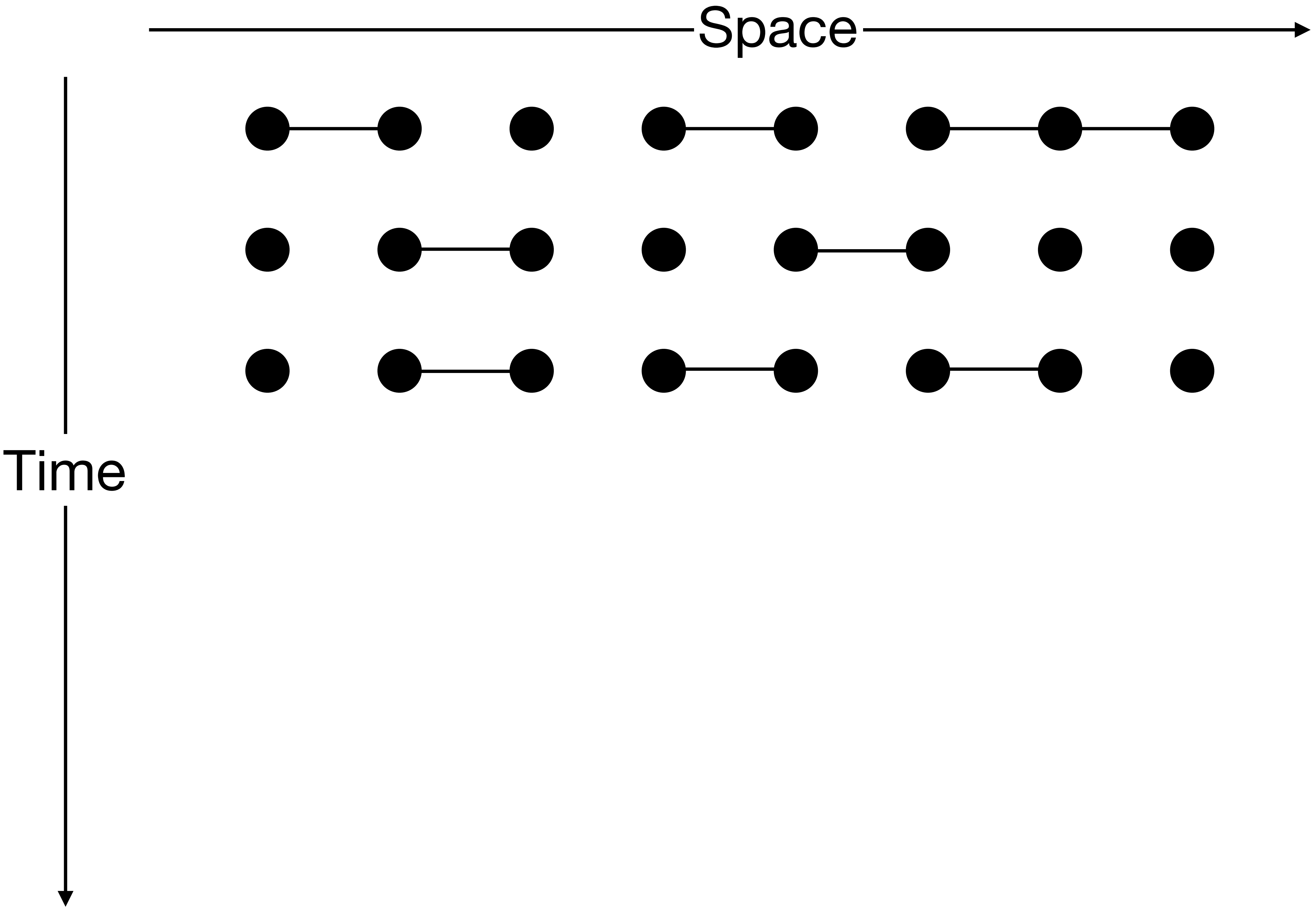
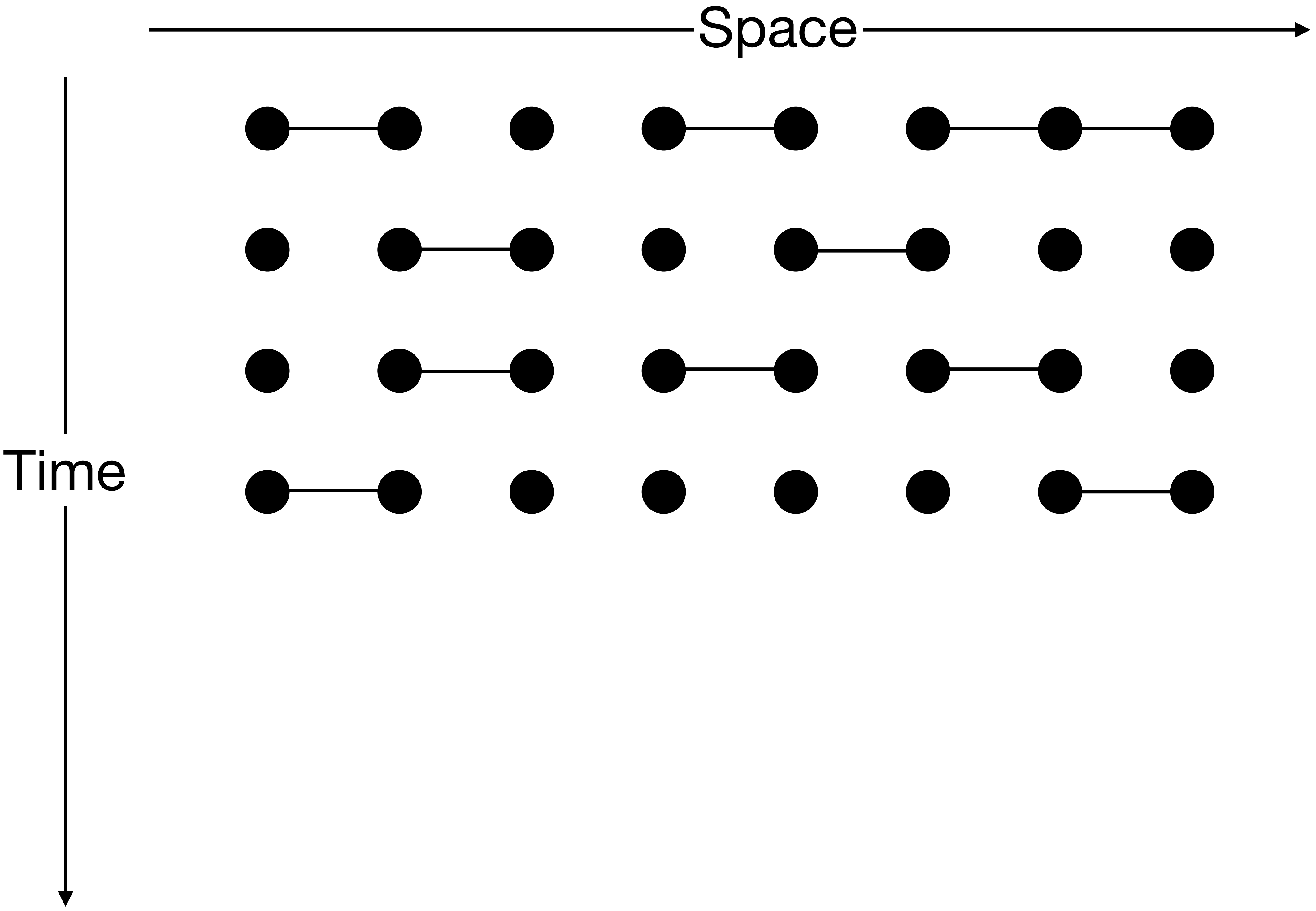


Evolving Graphs



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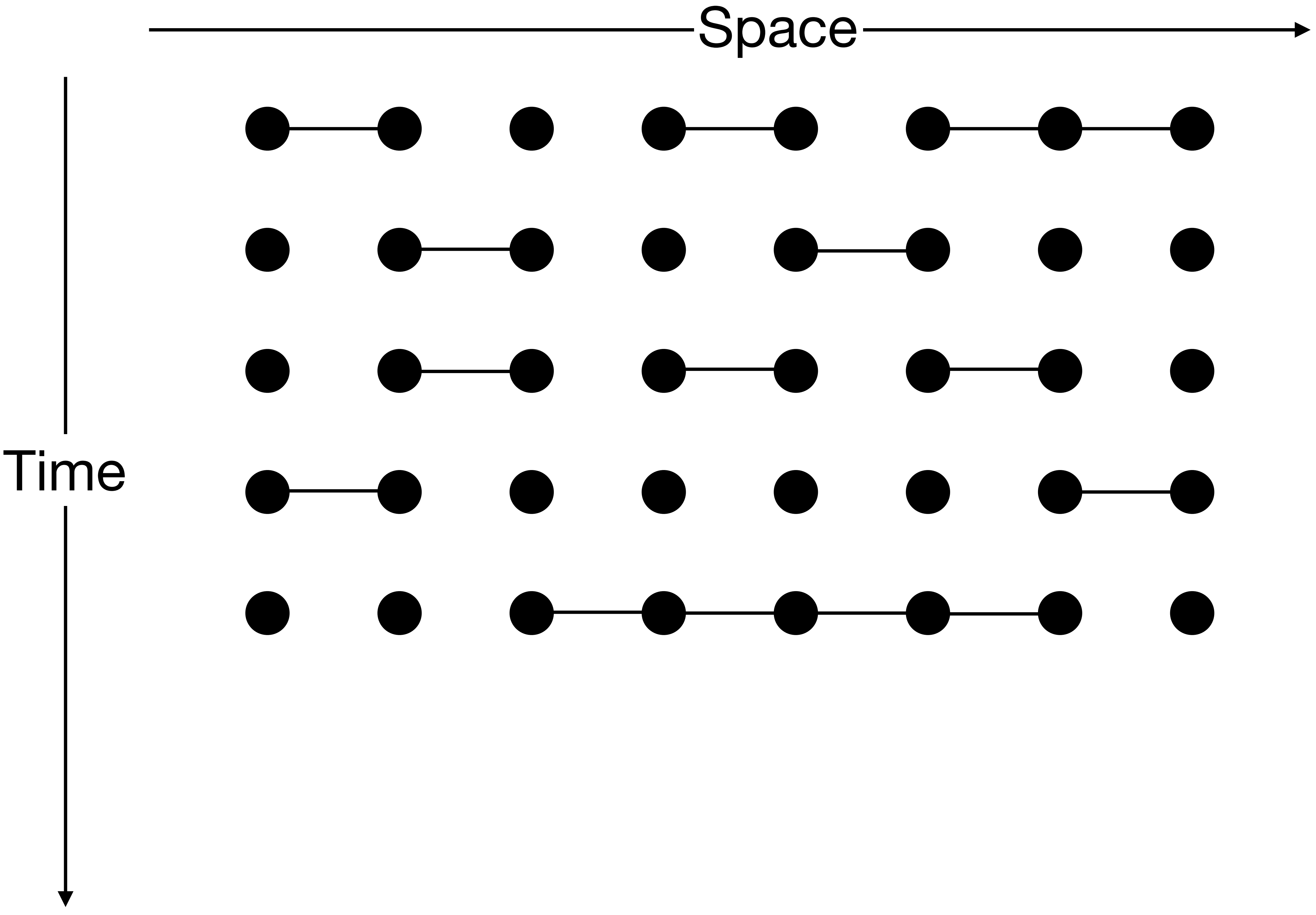
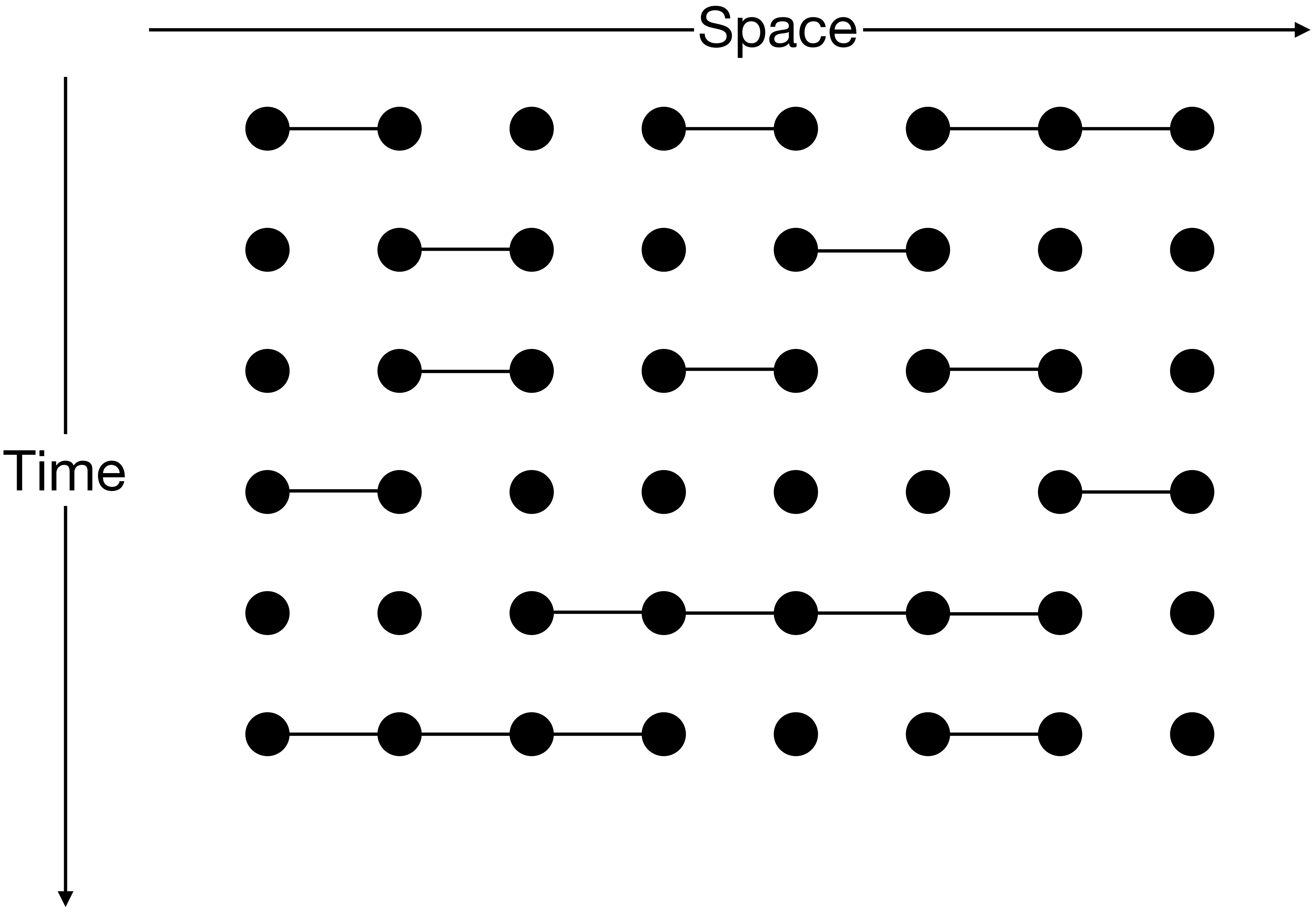
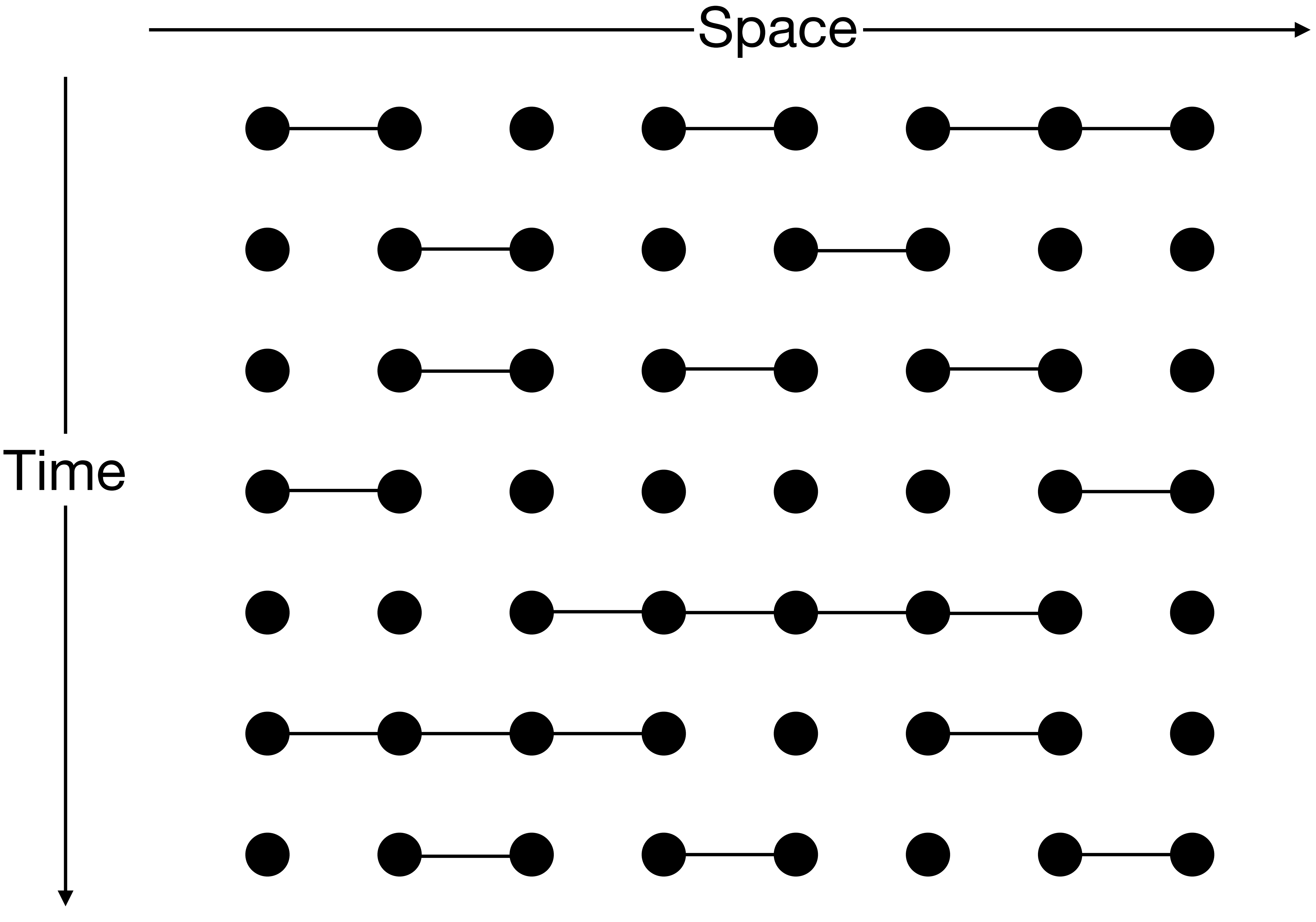


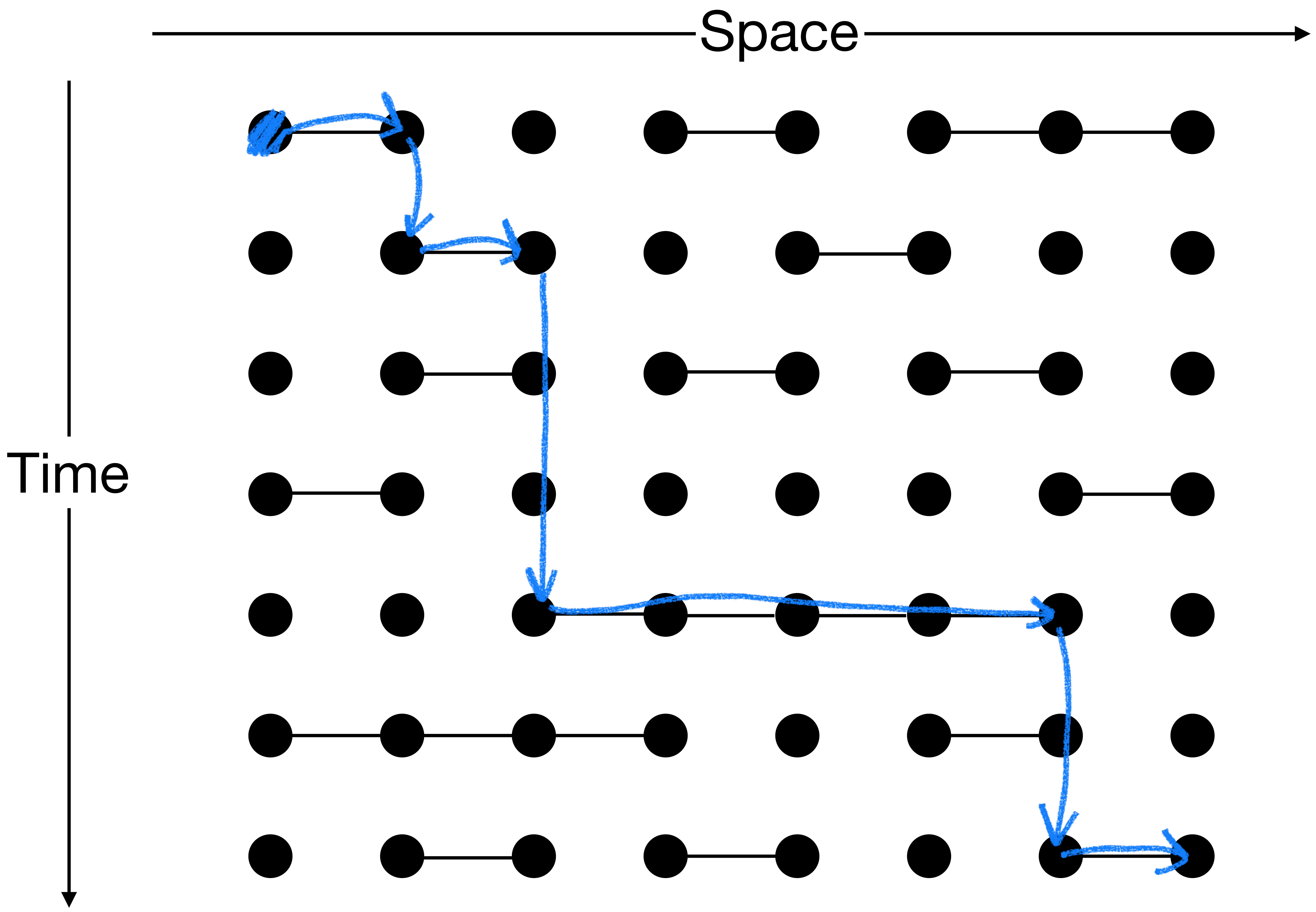
Evolving Graphs



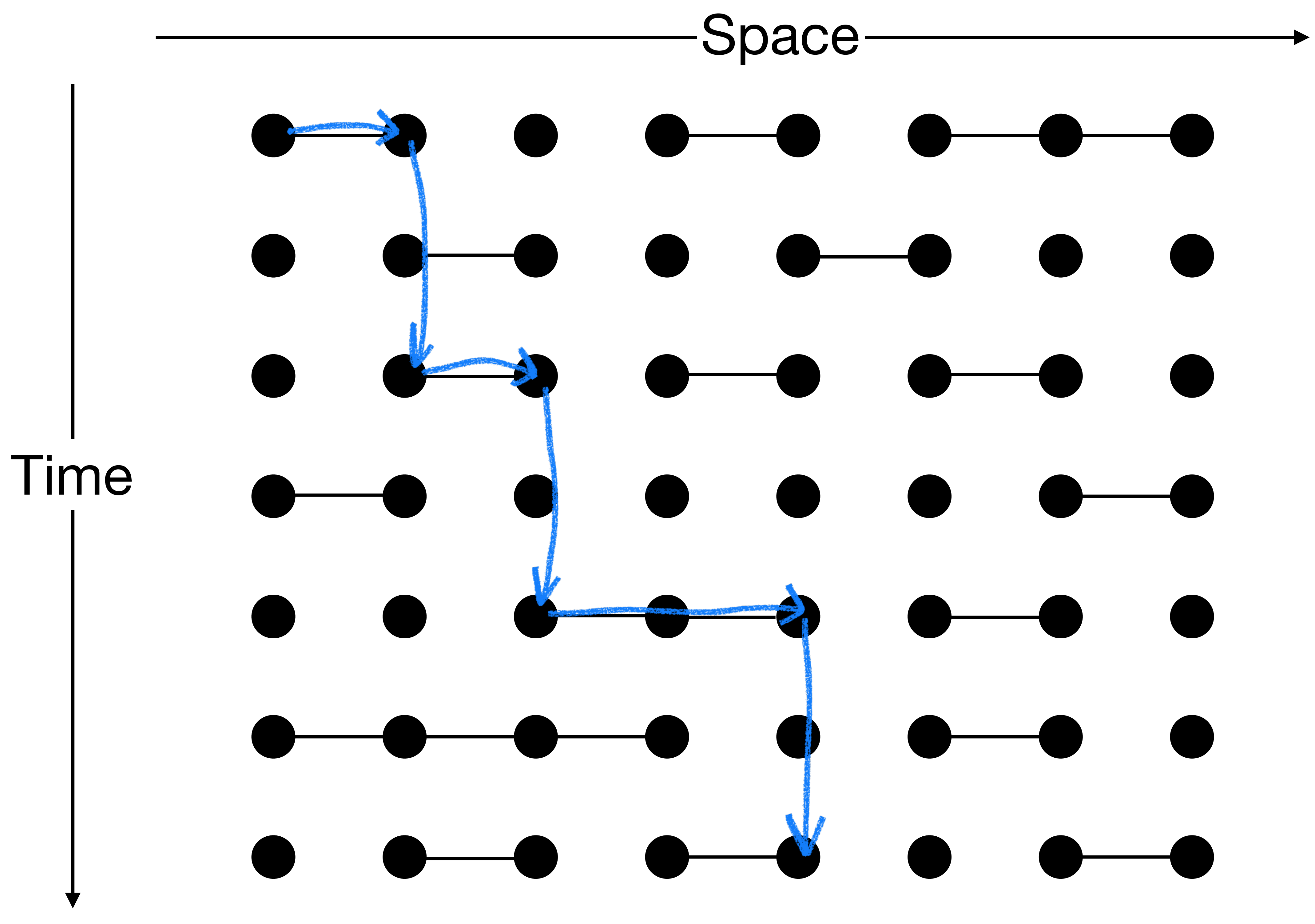
Evolving Graphs

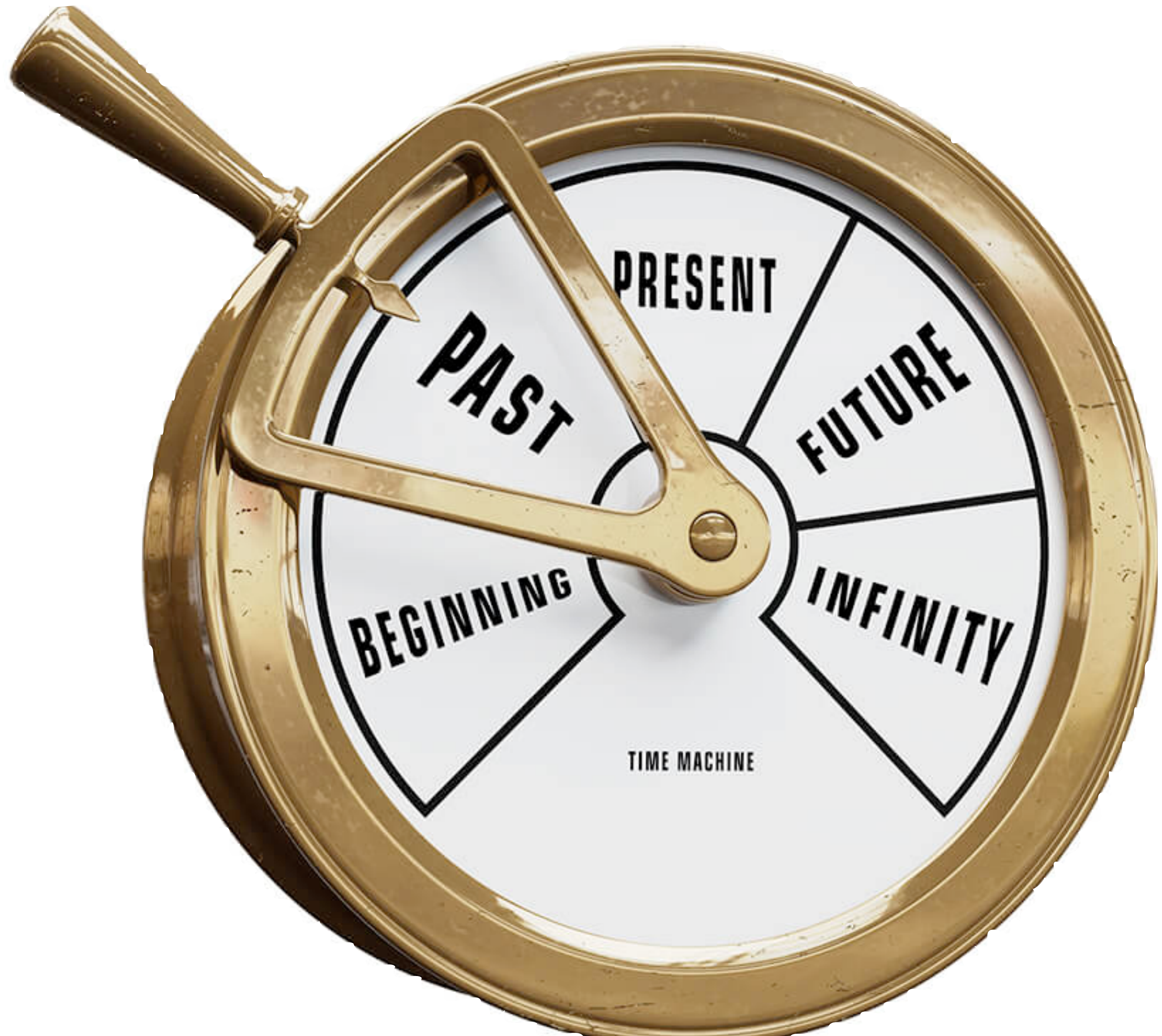


Journey

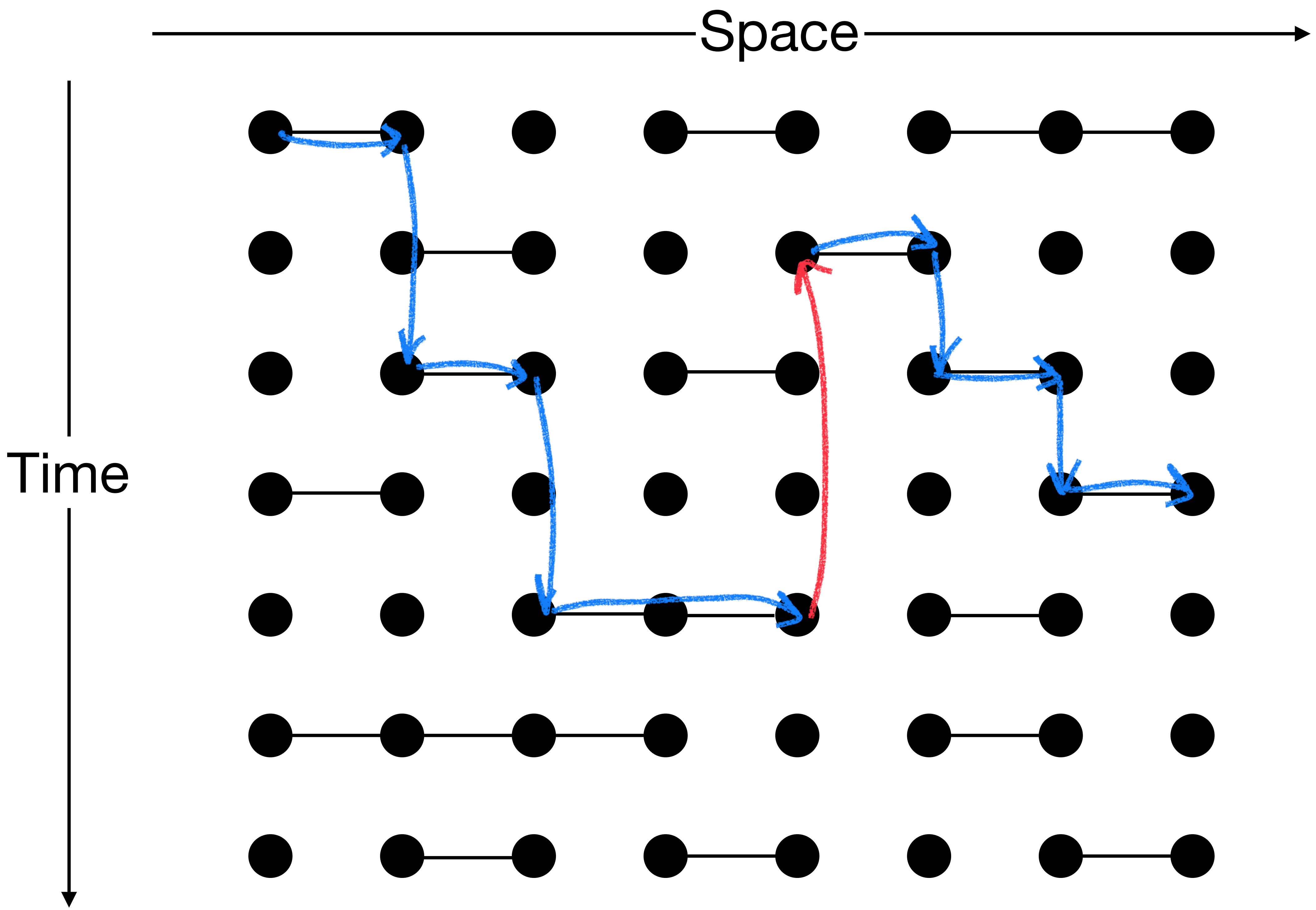


Journey

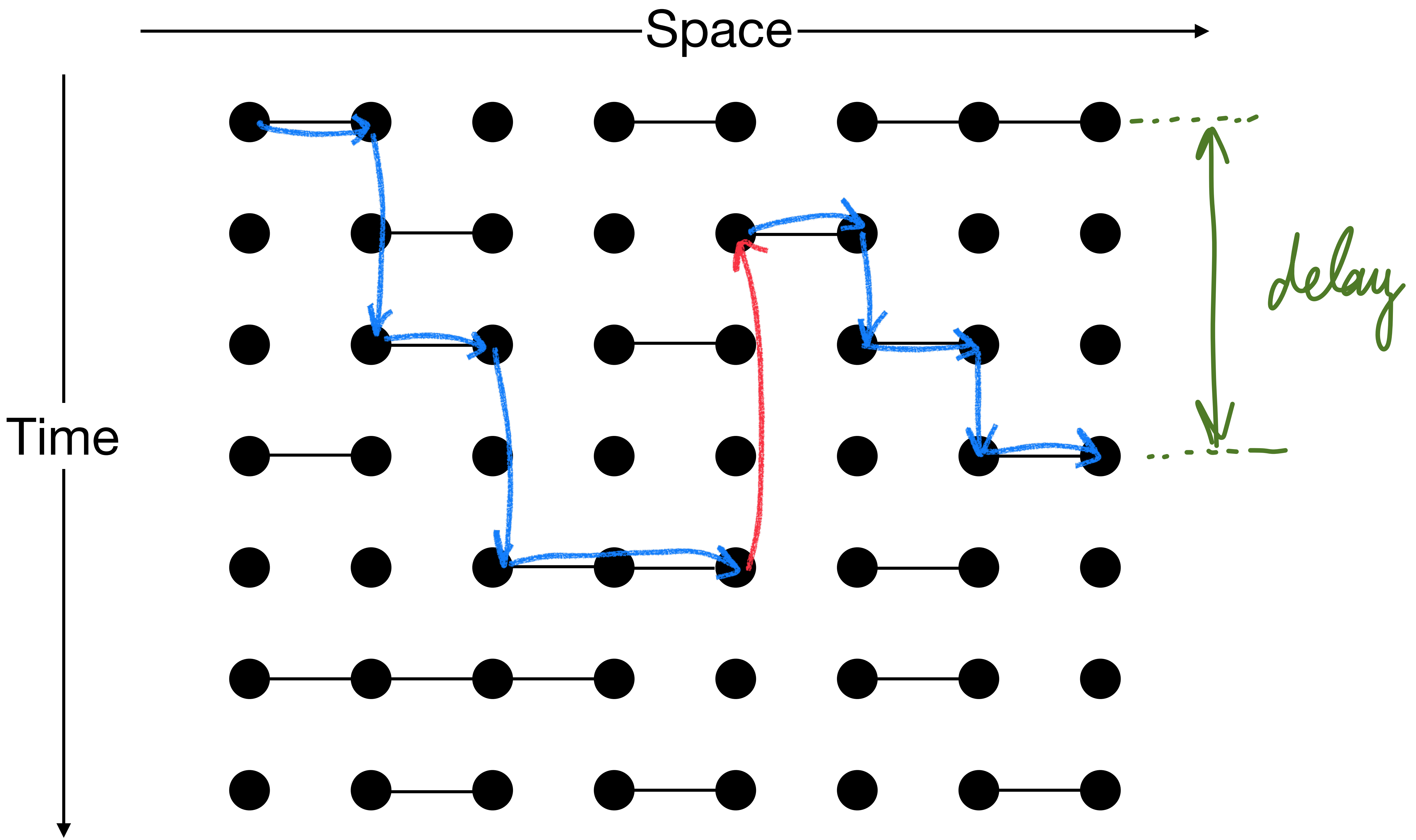



PRESENT
PAST
FUTURE
BEGINNING
INFINITY
TIME MACHINE

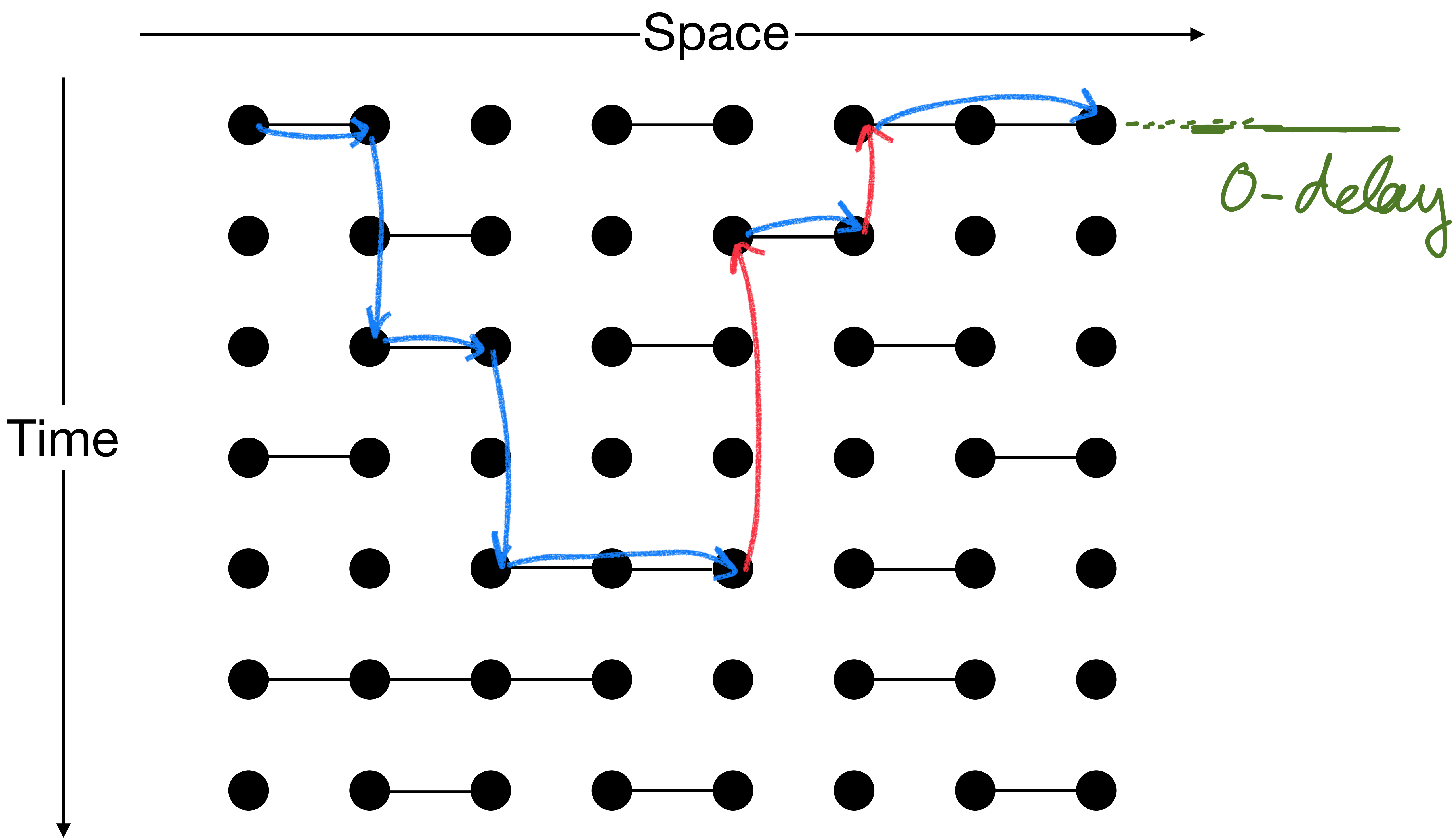
Space-Time Travel



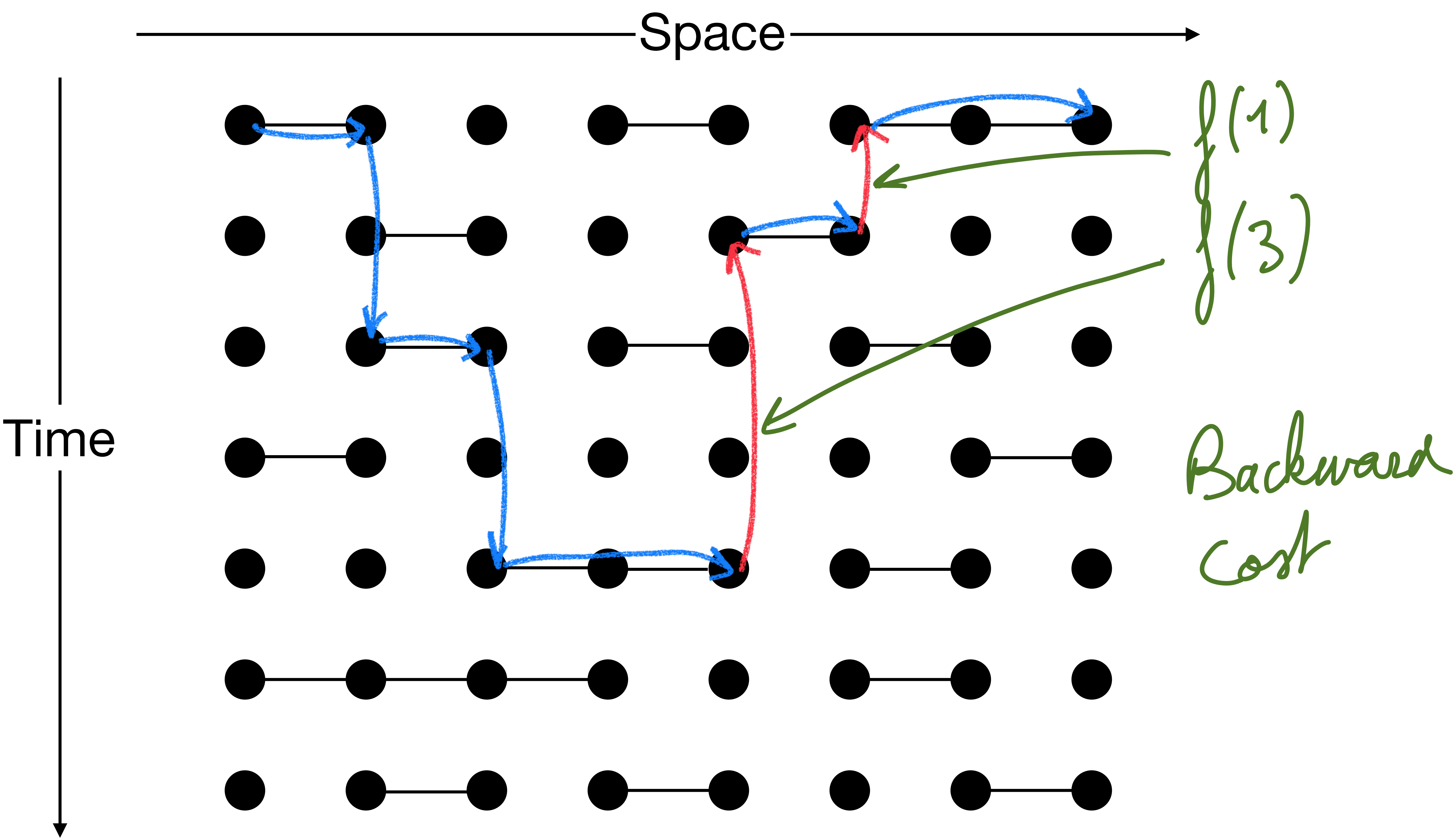
Travel Delay



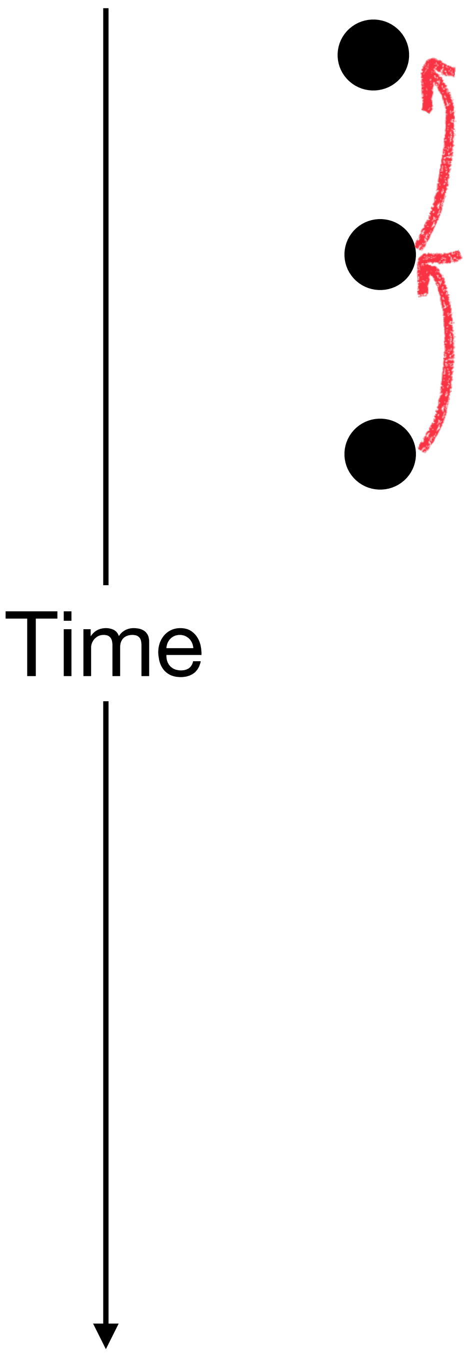
Travel Delay



Travel Cost



Cost function nice properties



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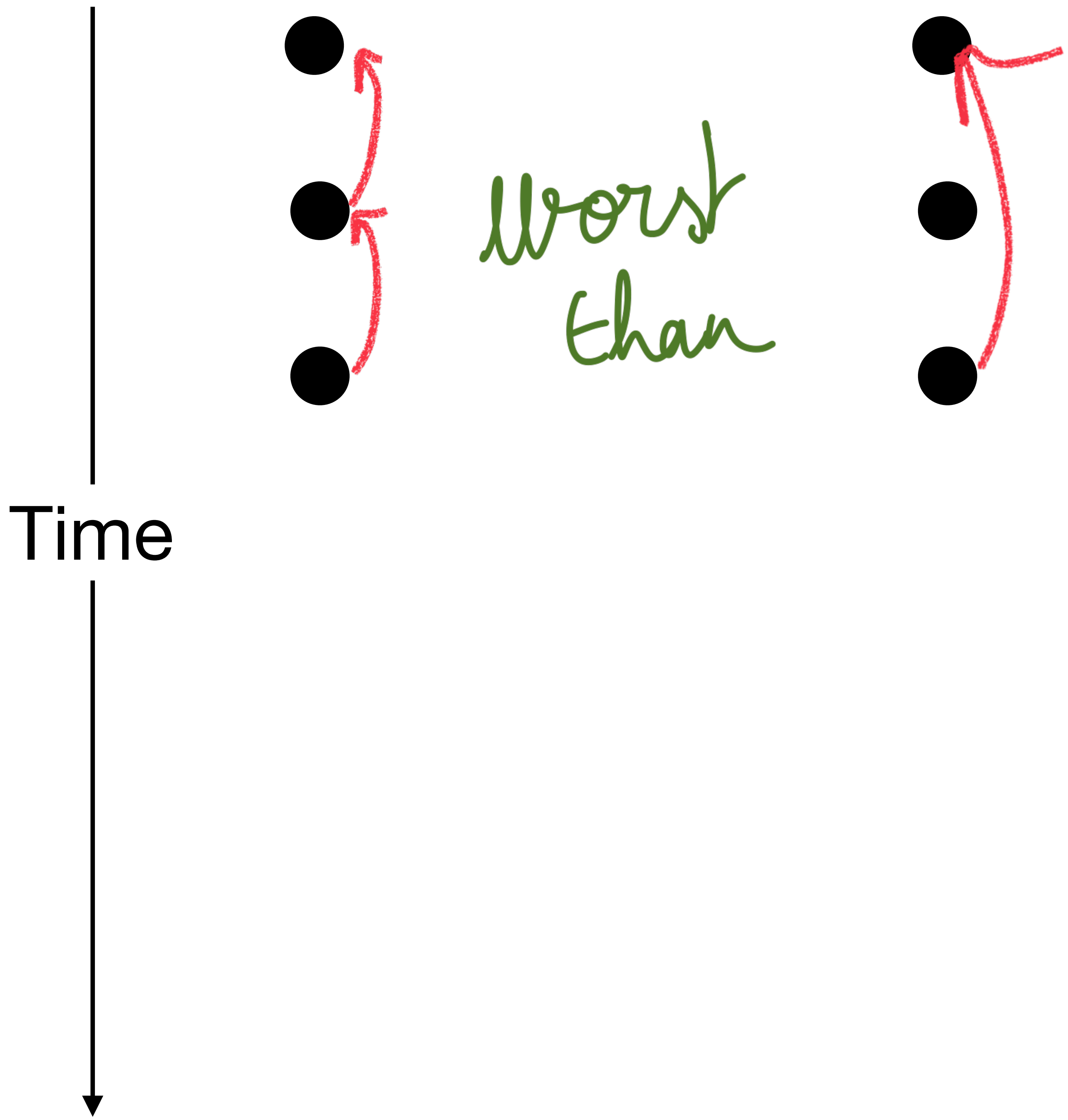
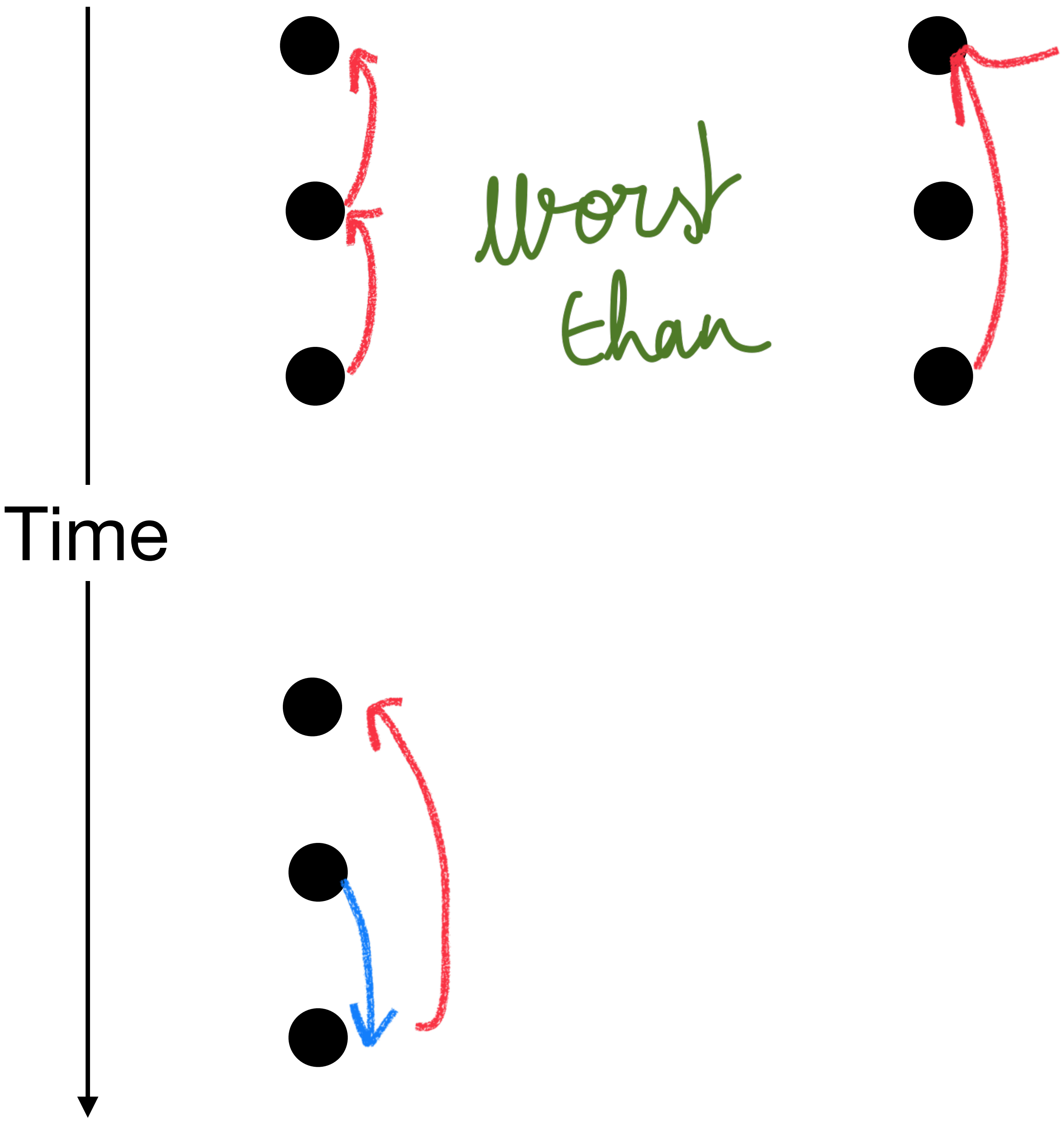


$$f(2) \leq f(1) + f(1)$$

$\Rightarrow$ Sub-additive

Cost function nice properties



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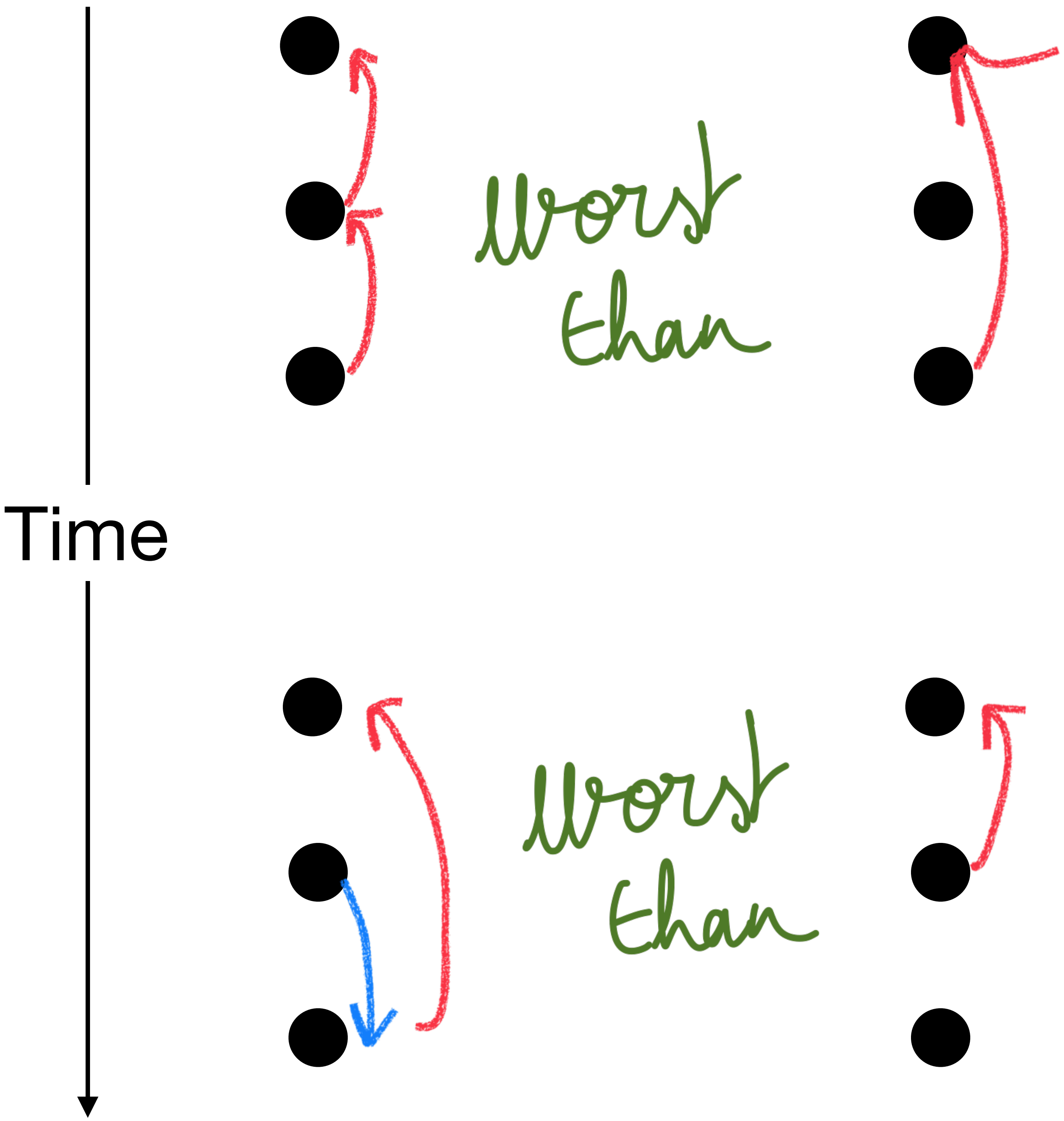


$f(2) \leq f(1) + f(1)$

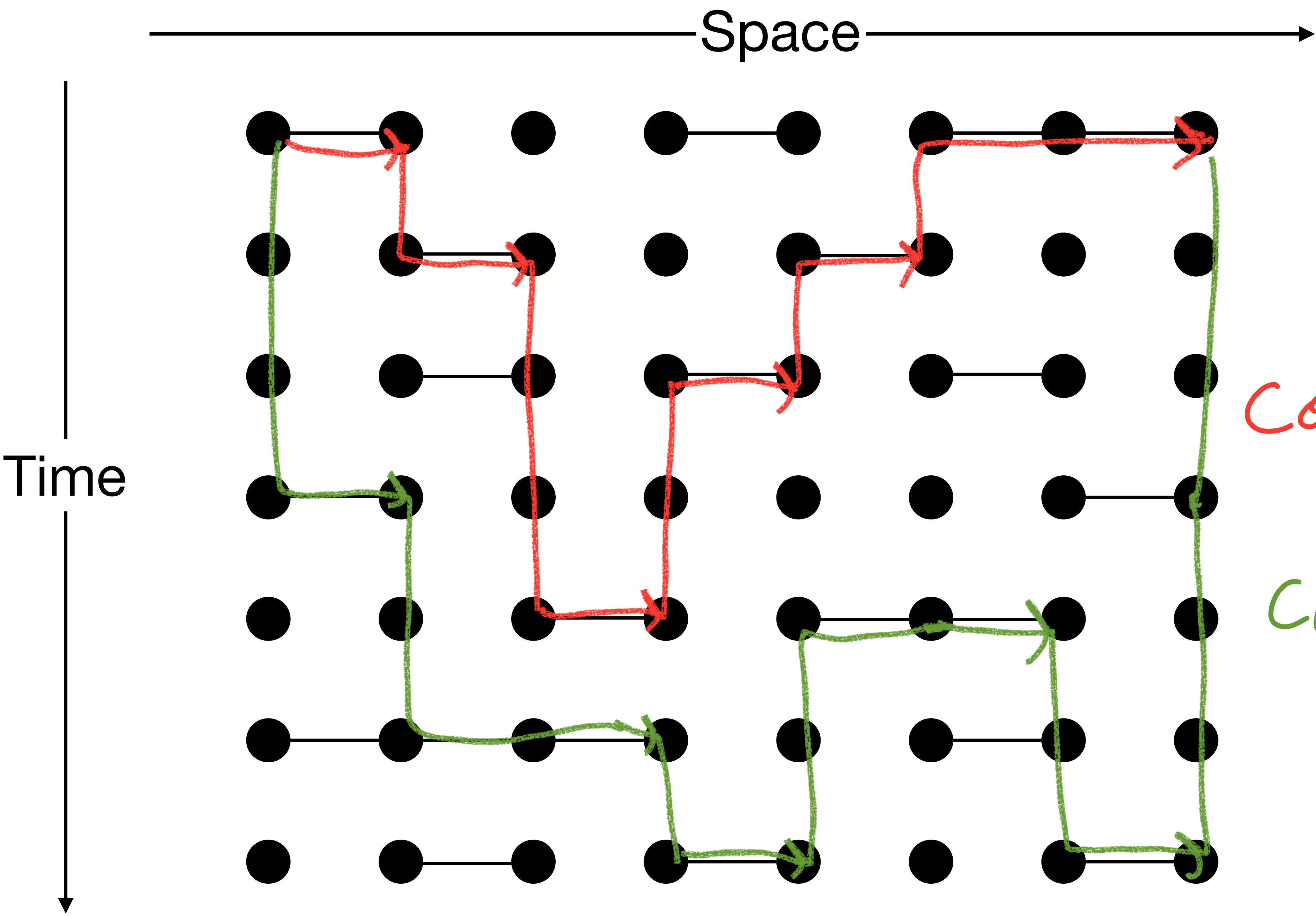
$\Rightarrow$ Sub-additive

$\Rightarrow$ non-decreasing

(and positive)

The ODOC Problem

$f : x \mapsto x$

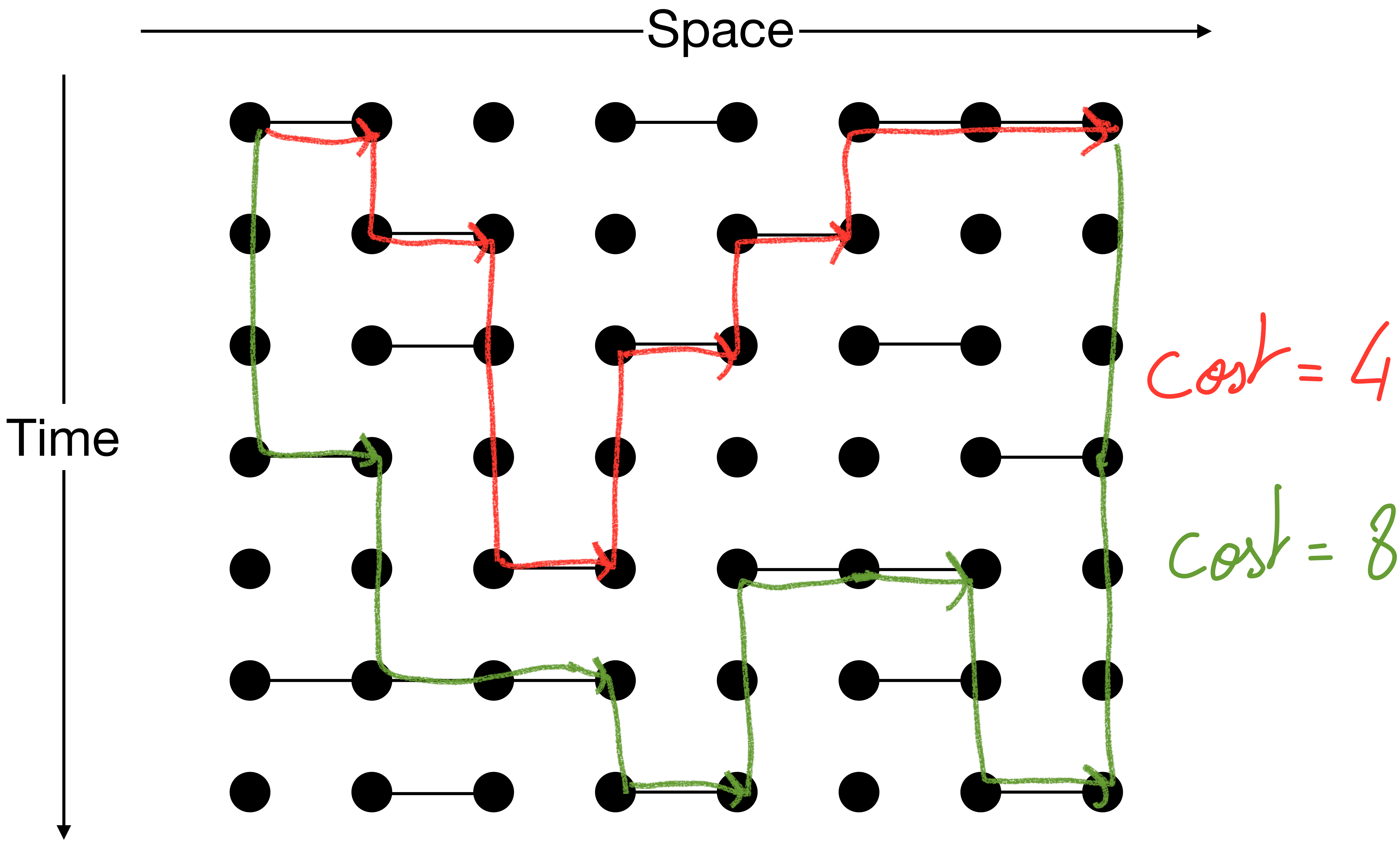


cost =

cost =

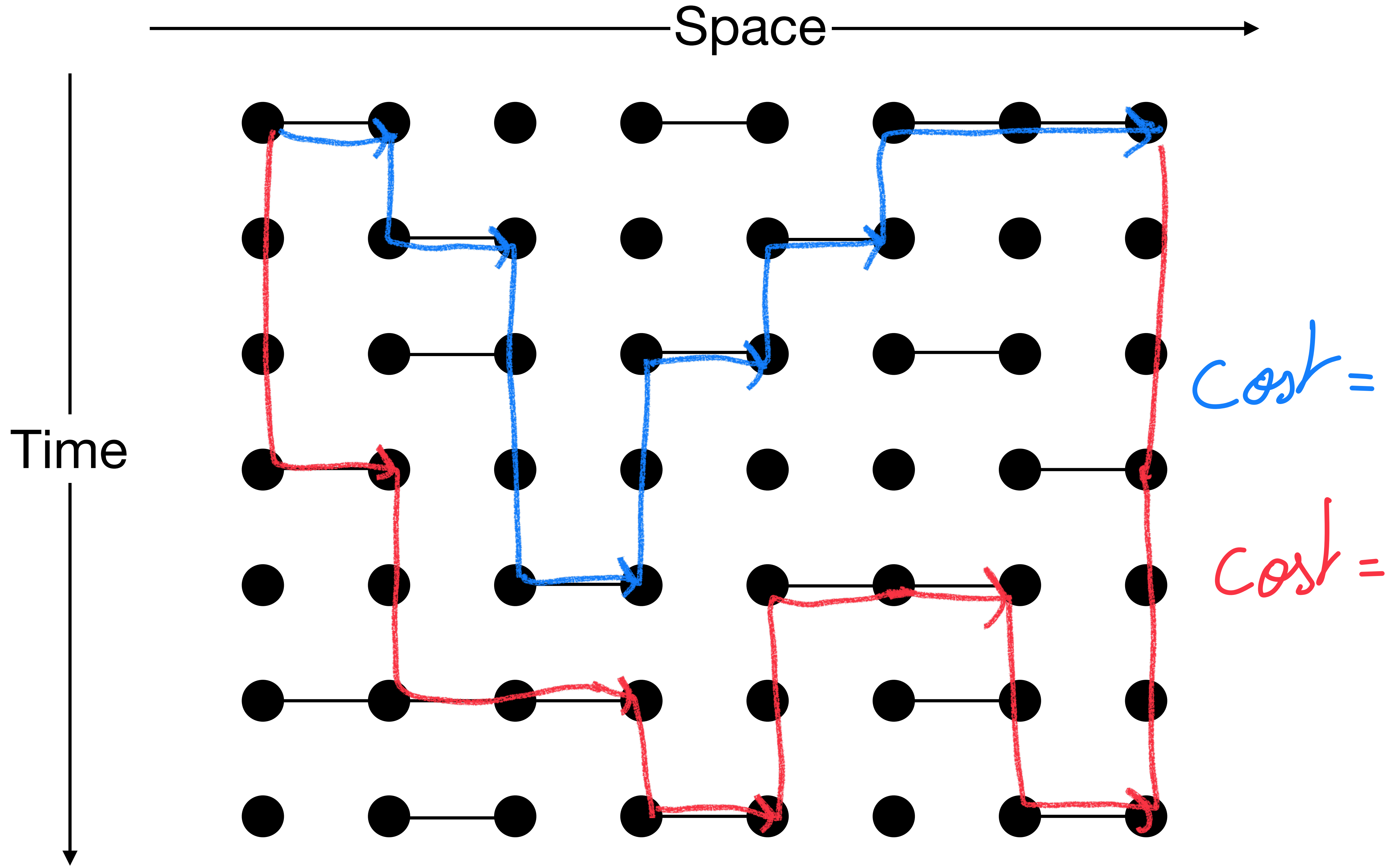
The ODOC Problem

$f : x \mapsto x$



The ODOC Problem

$f: x \mapsto 1$



The ODOC Problem

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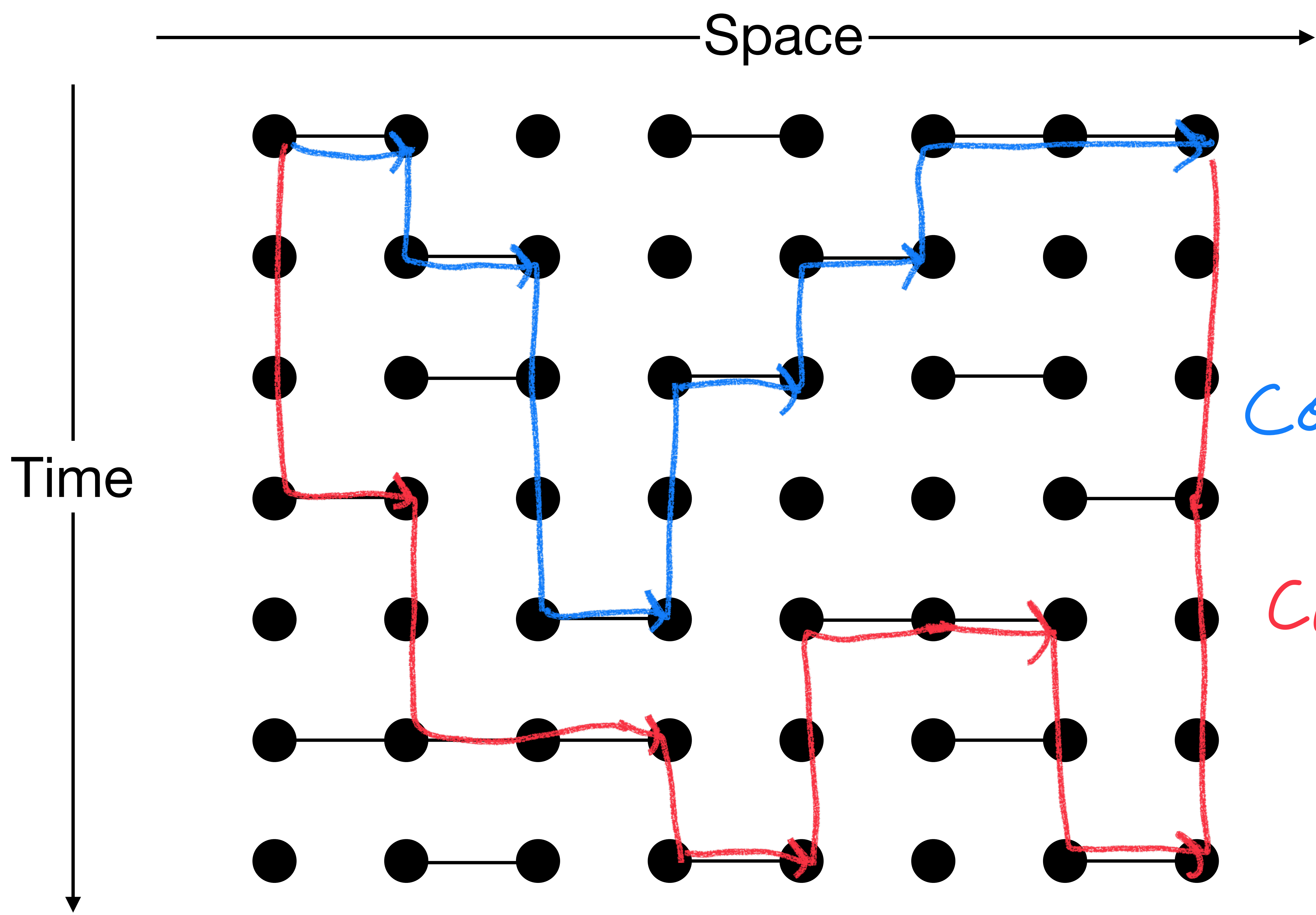


cost = 3

cost = 2

Related Work

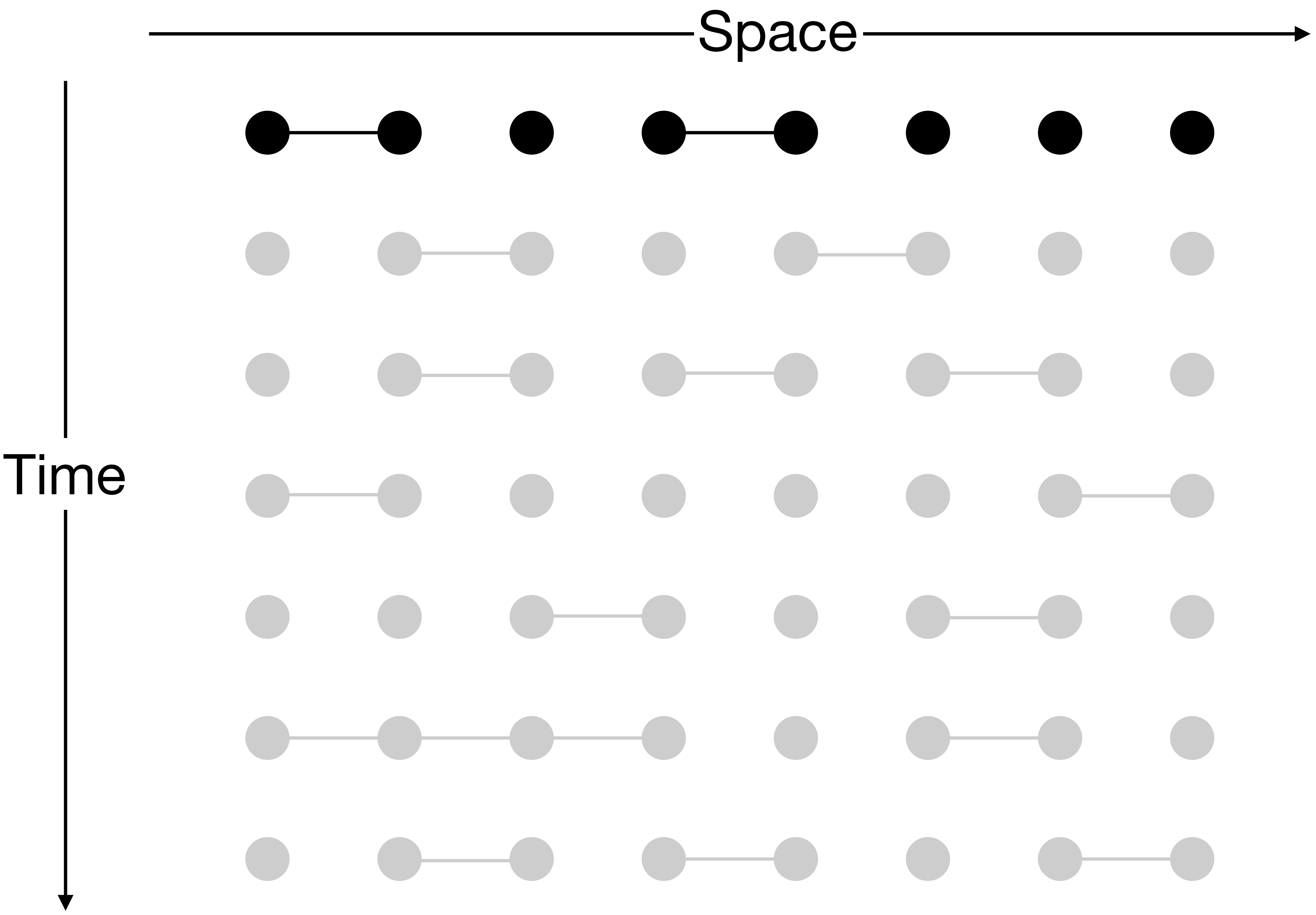
SSS 2023

Necessary condition on the cost function

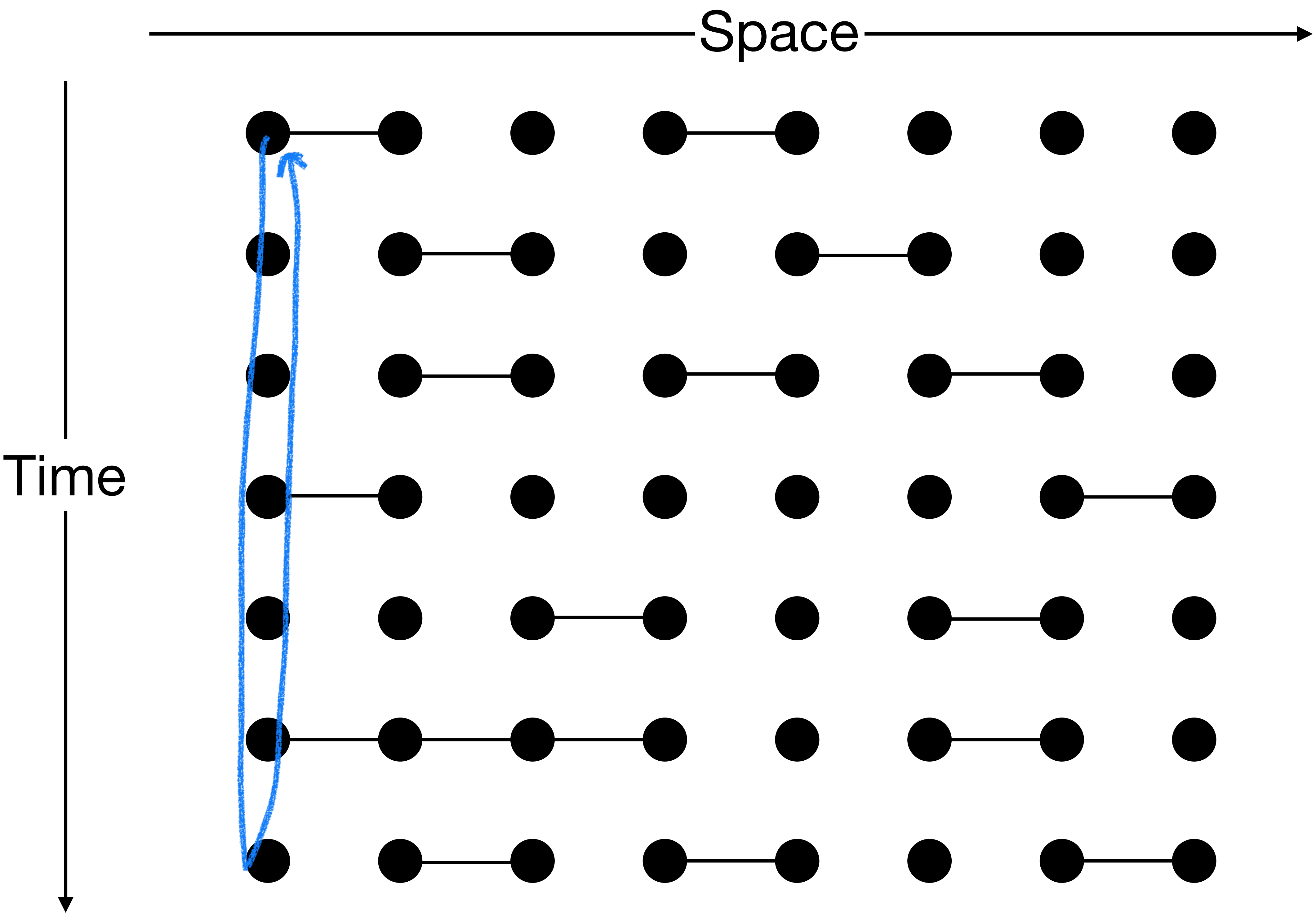
Offline ODOC problem is in P, even with a constraint on the cost, even in graphs evolving for infinity.

But, may create paradoxes!

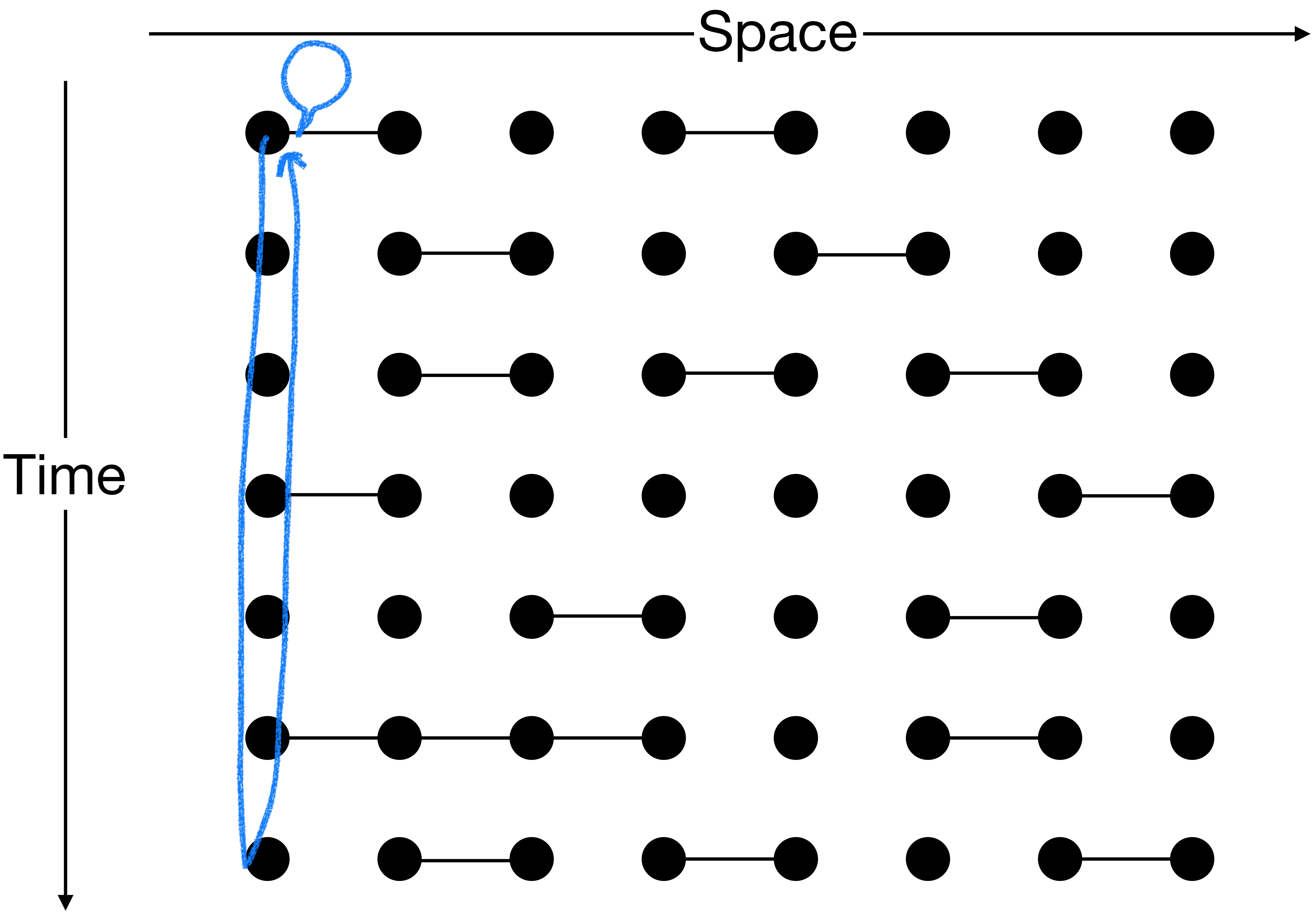
Learning paradoxe



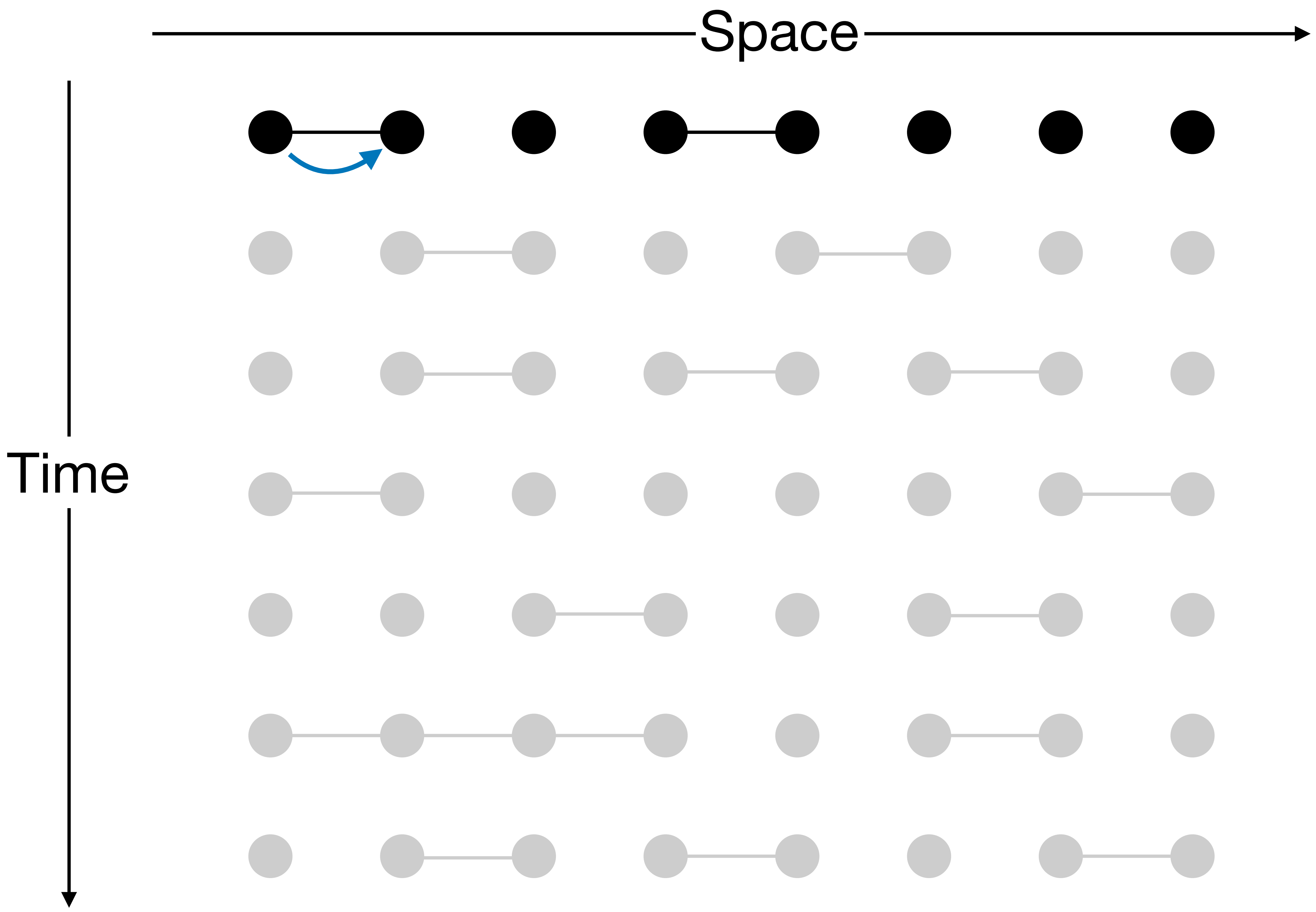
Learning paradoxe



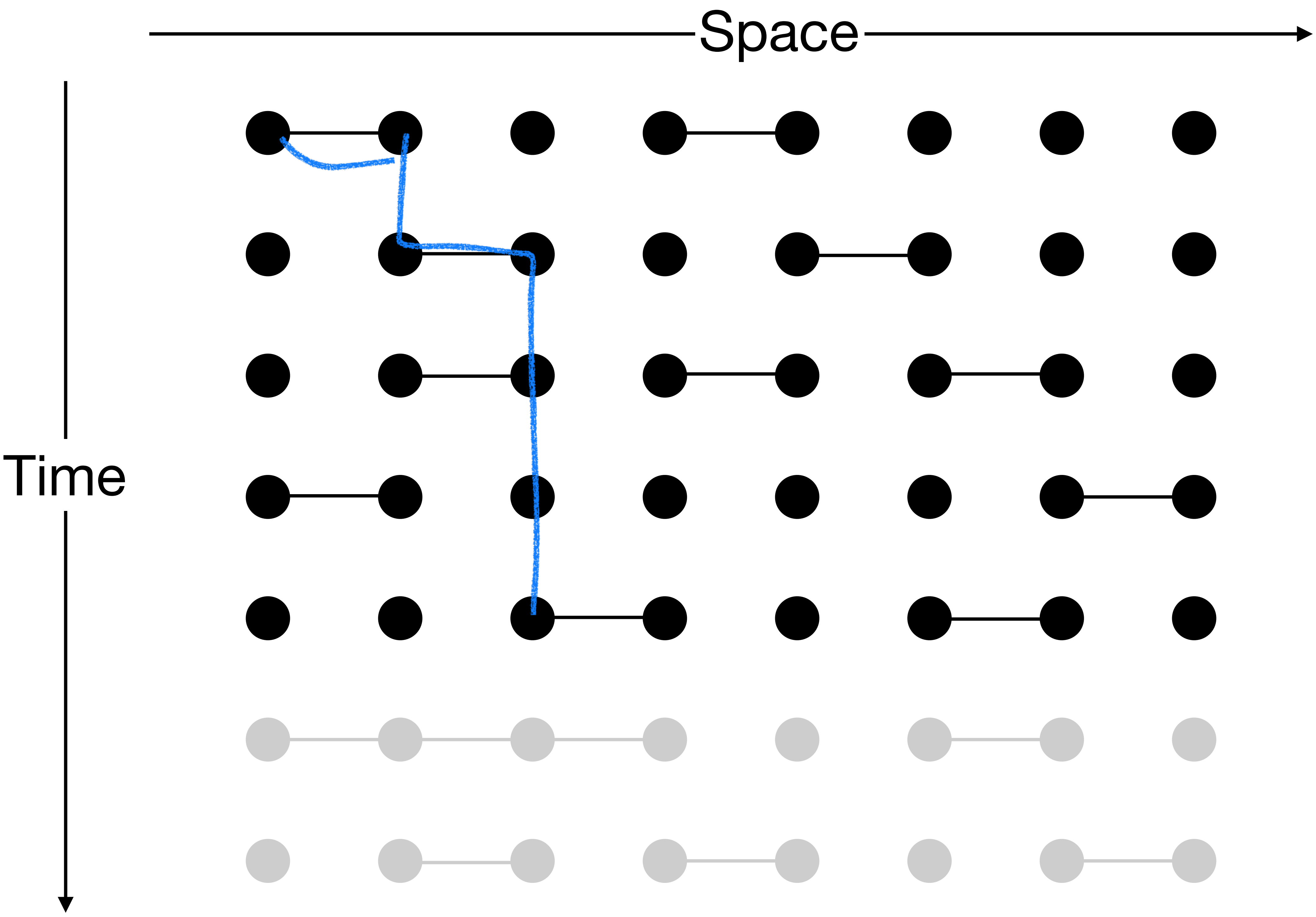
Learning paradoxe



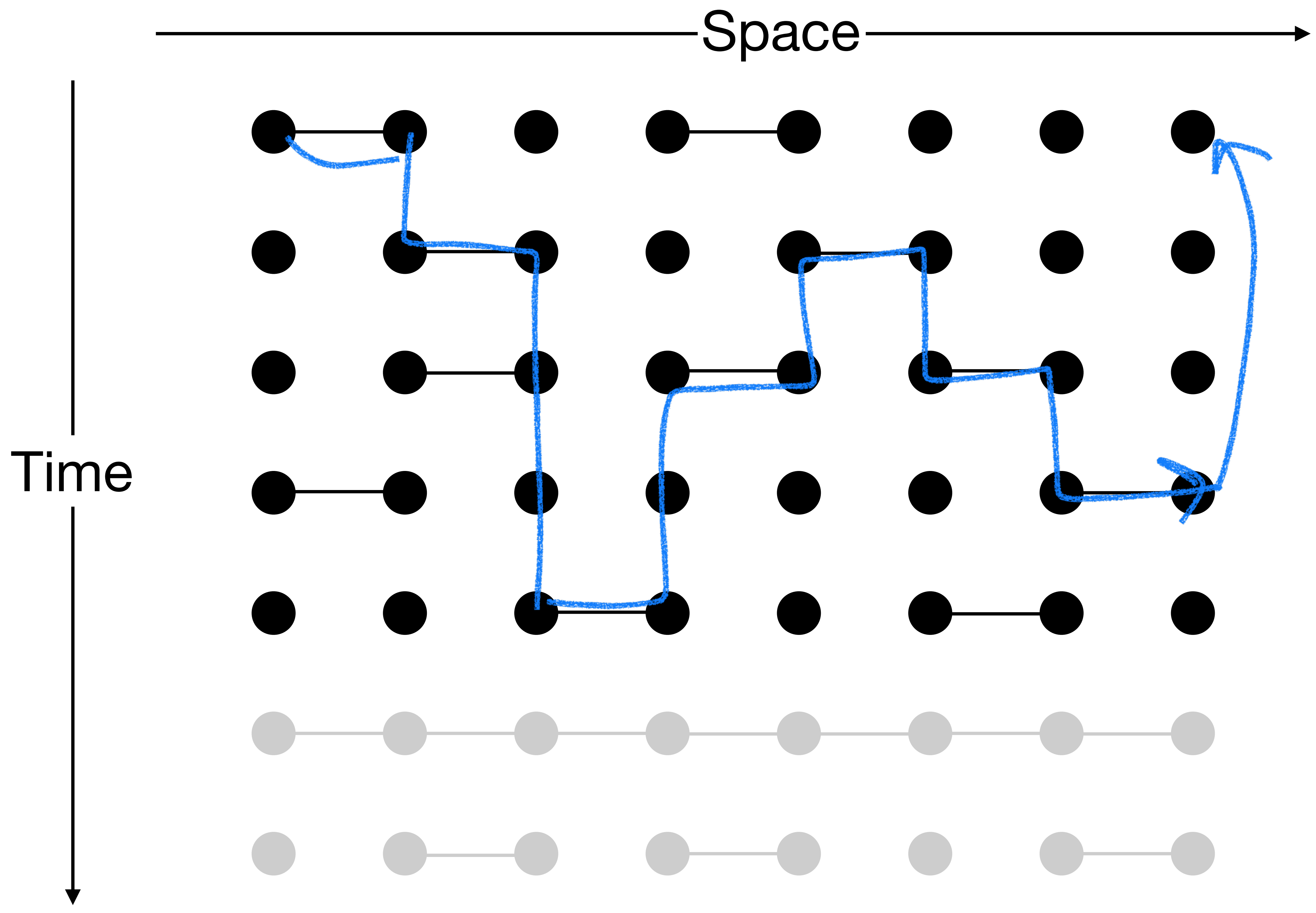
Time-Online Algorithm



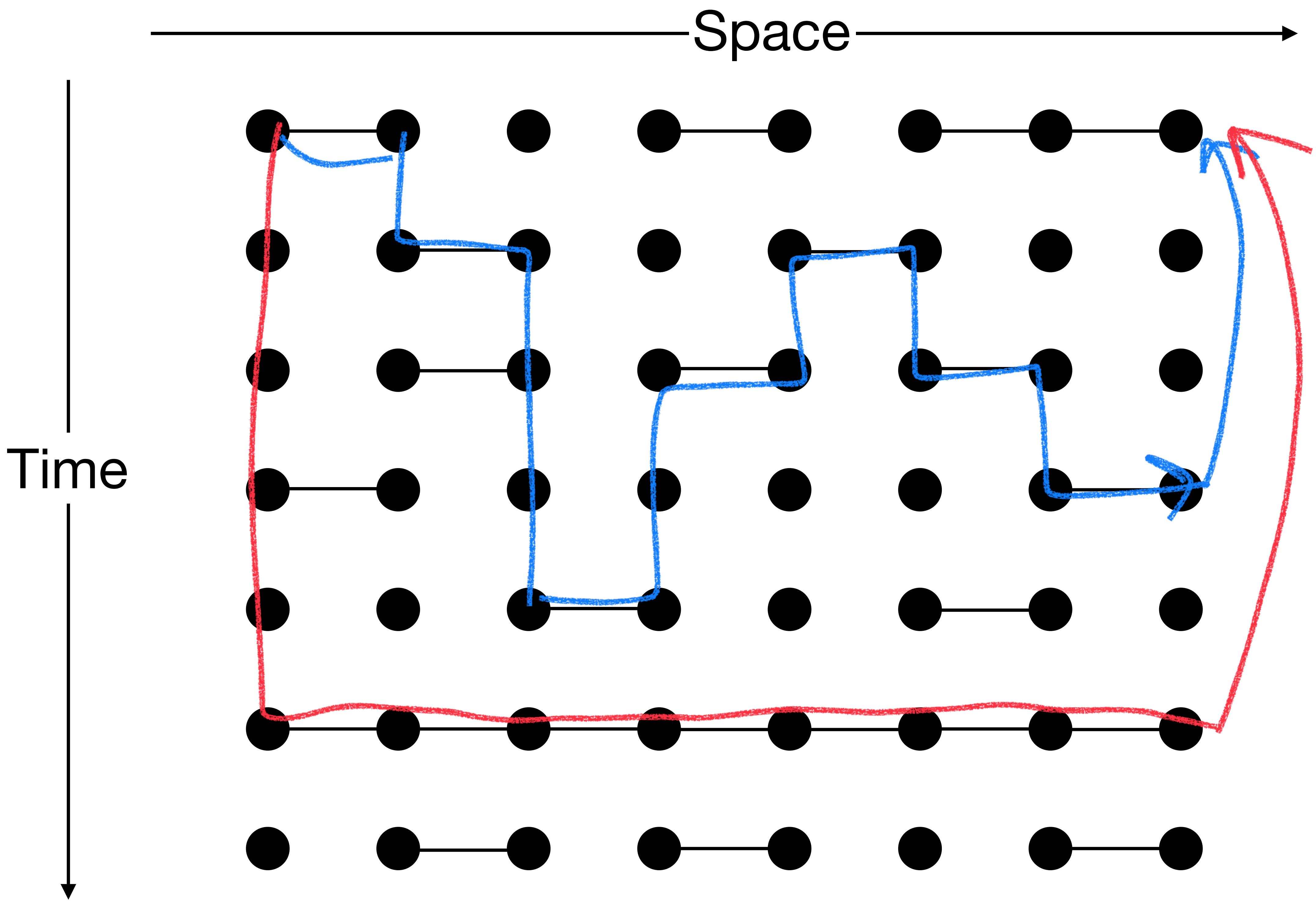
Time-Online Algorithm



Time-Online Algorithm



Competitive Ratio



Competitive Ratio

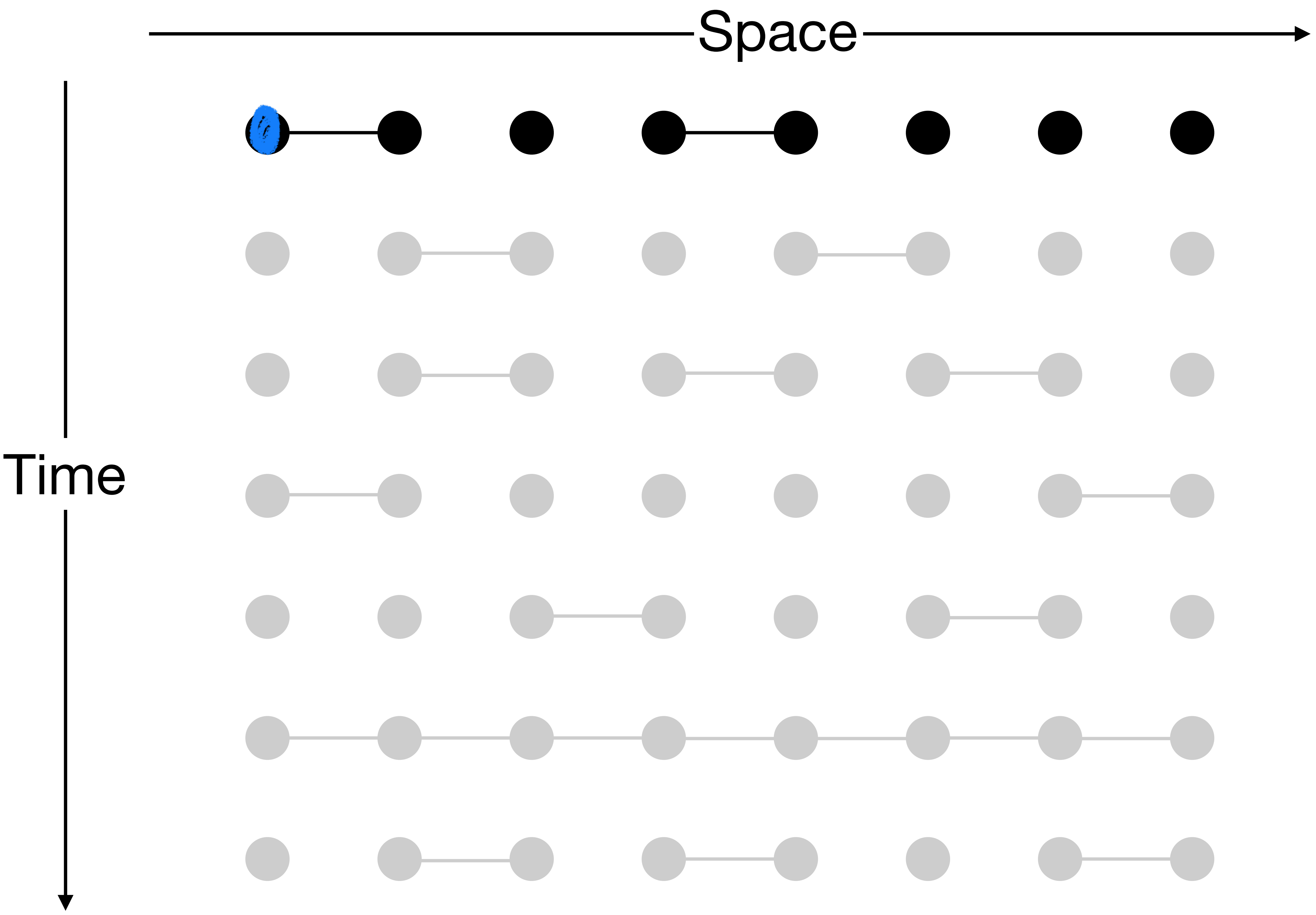
A is か-competitive if

Competitive Ratio

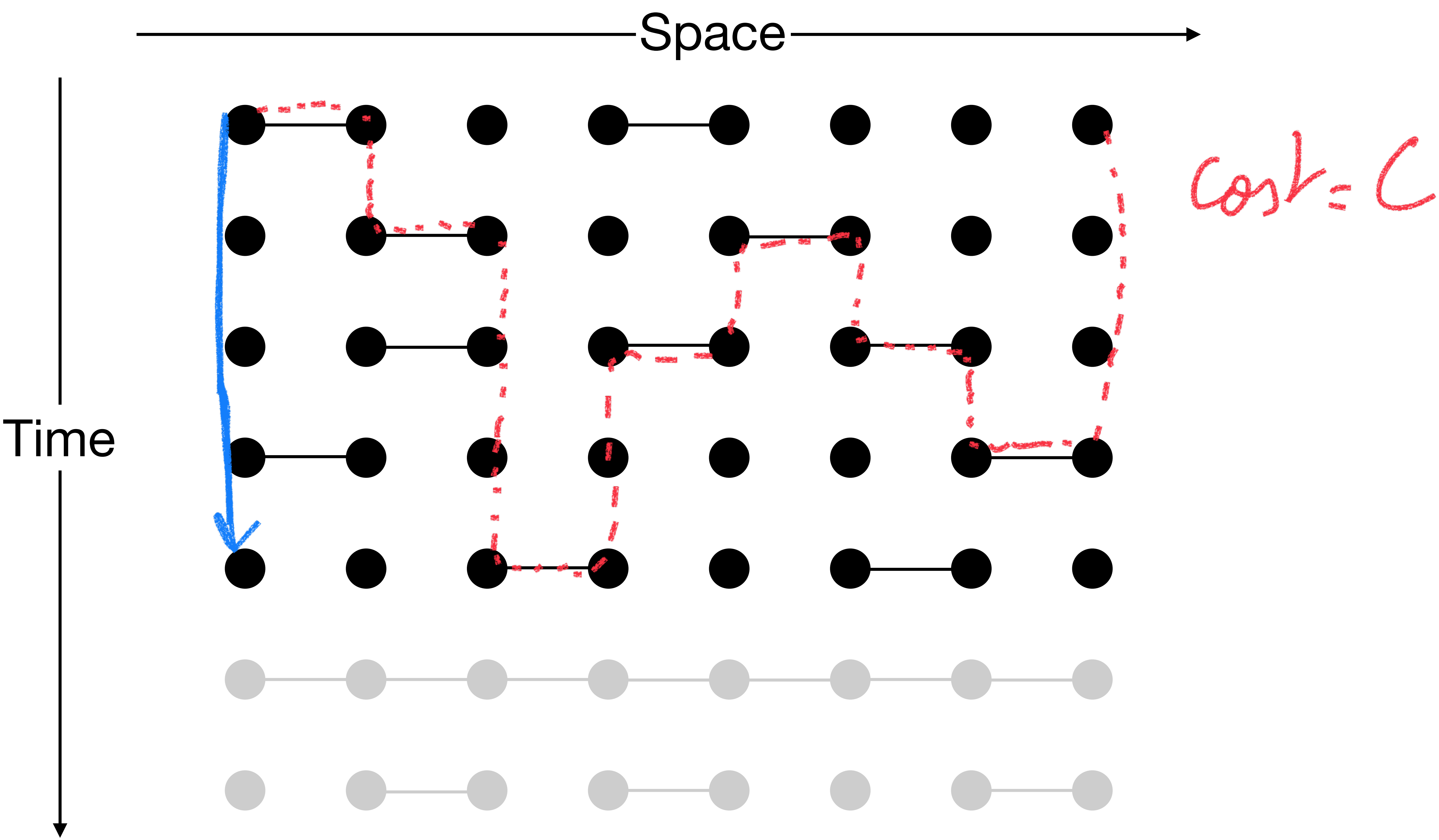
A is か-competitive if

$$\forall G, \quad \frac{Cost(A, G)}{Cost(Opt, G)} \leq \text{か}$$

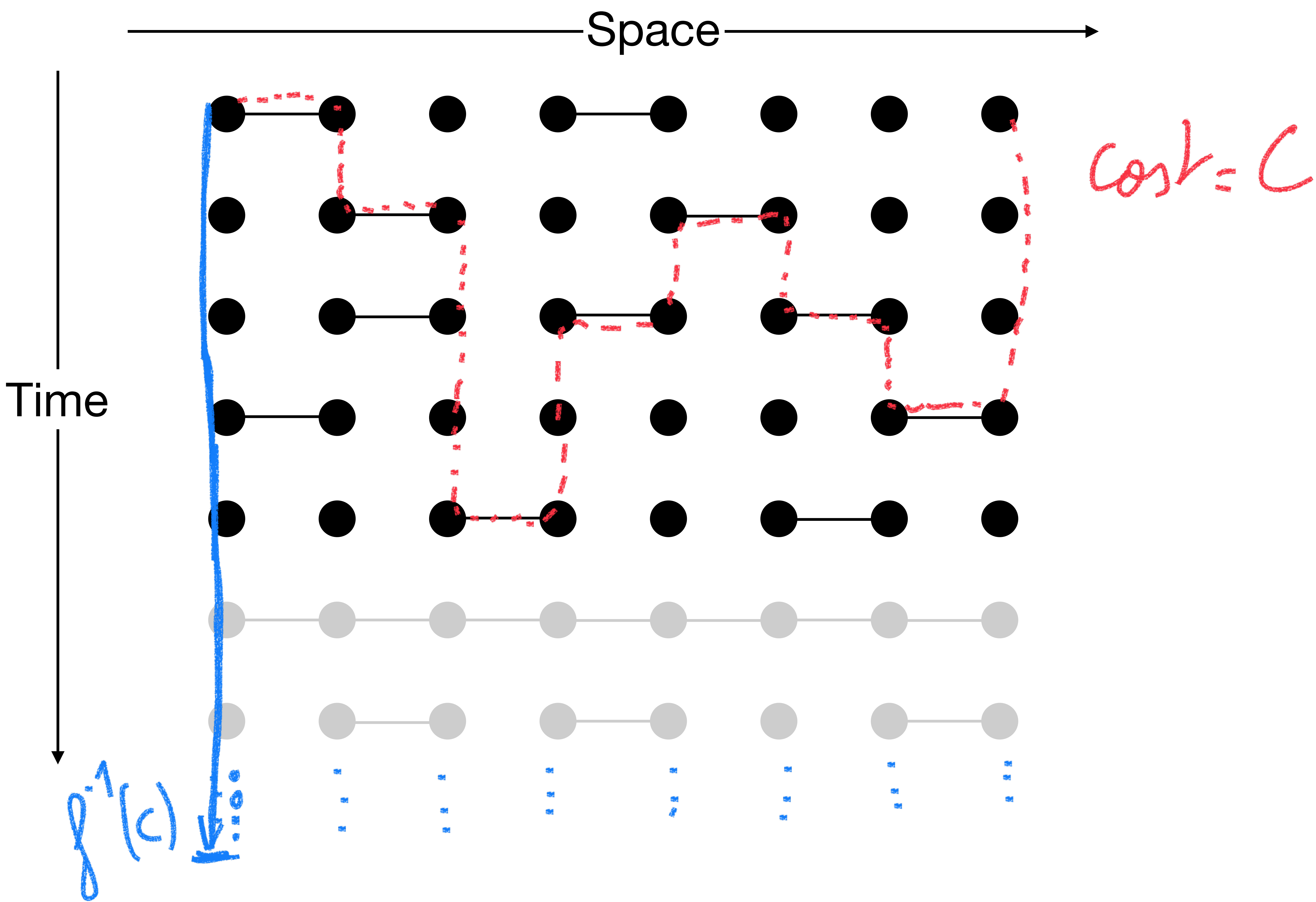
Time-Online Algorithm



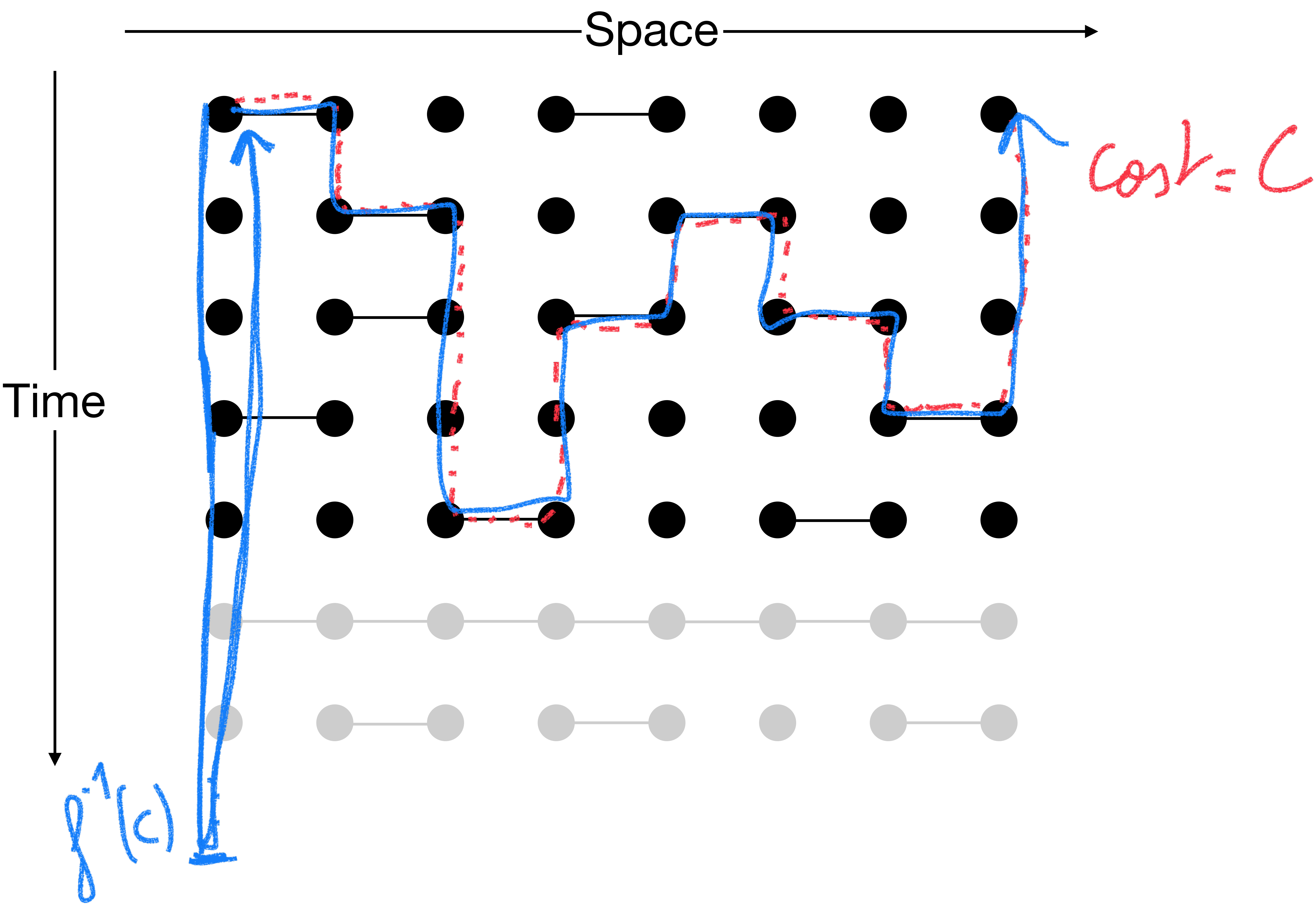
Time-Online Algorithm



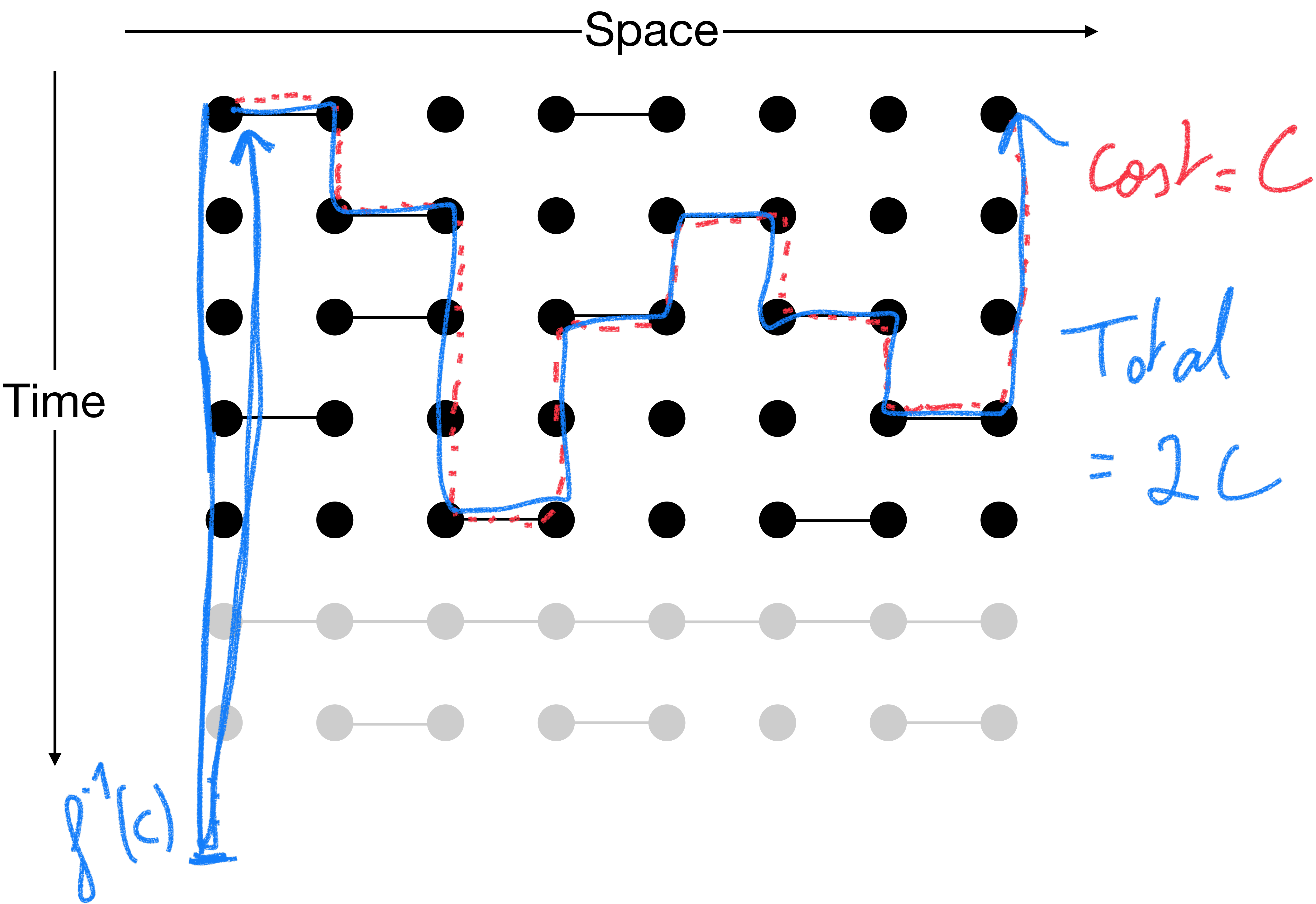
Time-Online Algorithm



Time-Online Algorithm



Time-Online Algorithm



Time-Online Algorithm

The following algorithm is 2-competitive

for any f with $\lim_{x \to \infty} f(x) = \infty$

Algorithm 1 T-Online Algorithm

Input :

G': the known evolving graph

time: the current time

u: the current node (here u is always the starting node s)

Let $T_{\max}$ be an optimal space-time travel in G', if it exists, starting at time 0, from node s to node d.

if $T_{\max}$ *does not exist* ***or*** $\mathfrak{f}(time + 1) < cost(T_{\max})$ **then**

wait 1 time instant;

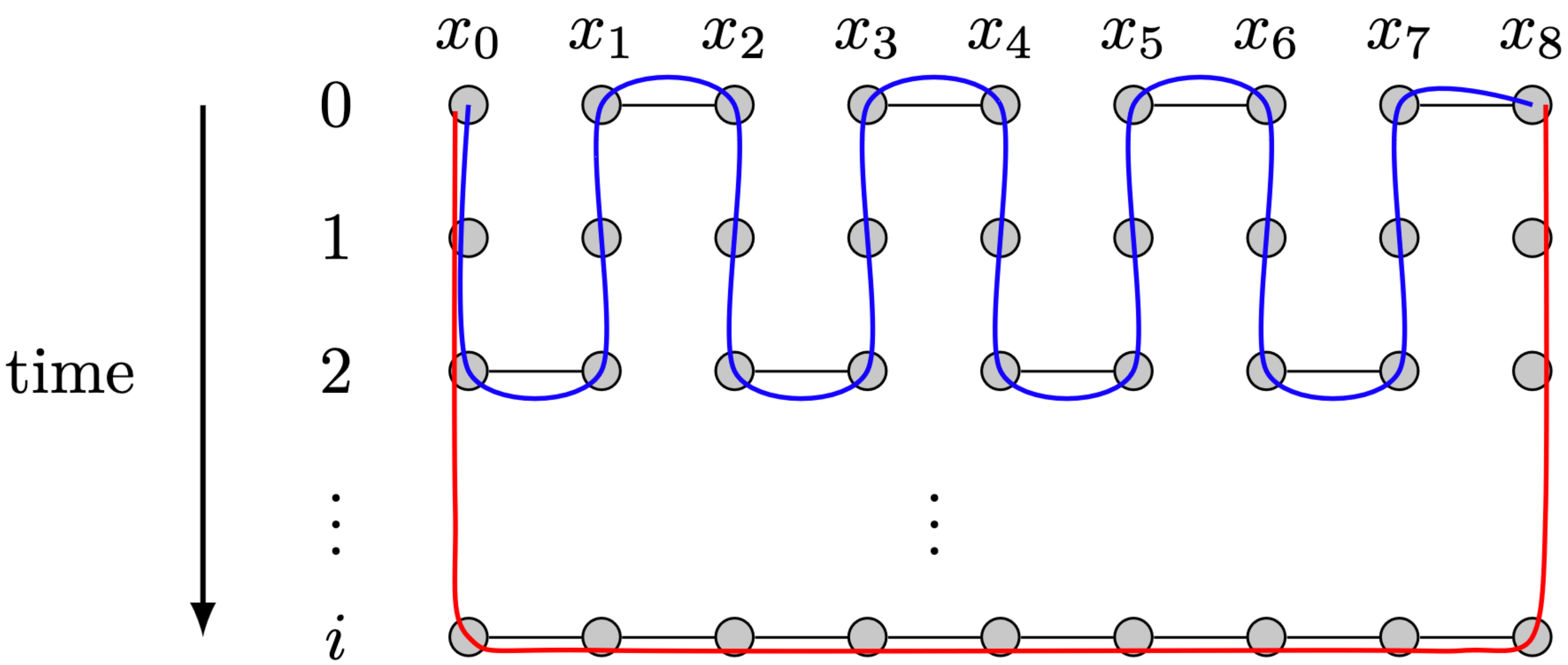
else

go back at time 0, then follow the travel $T_{\max}$;

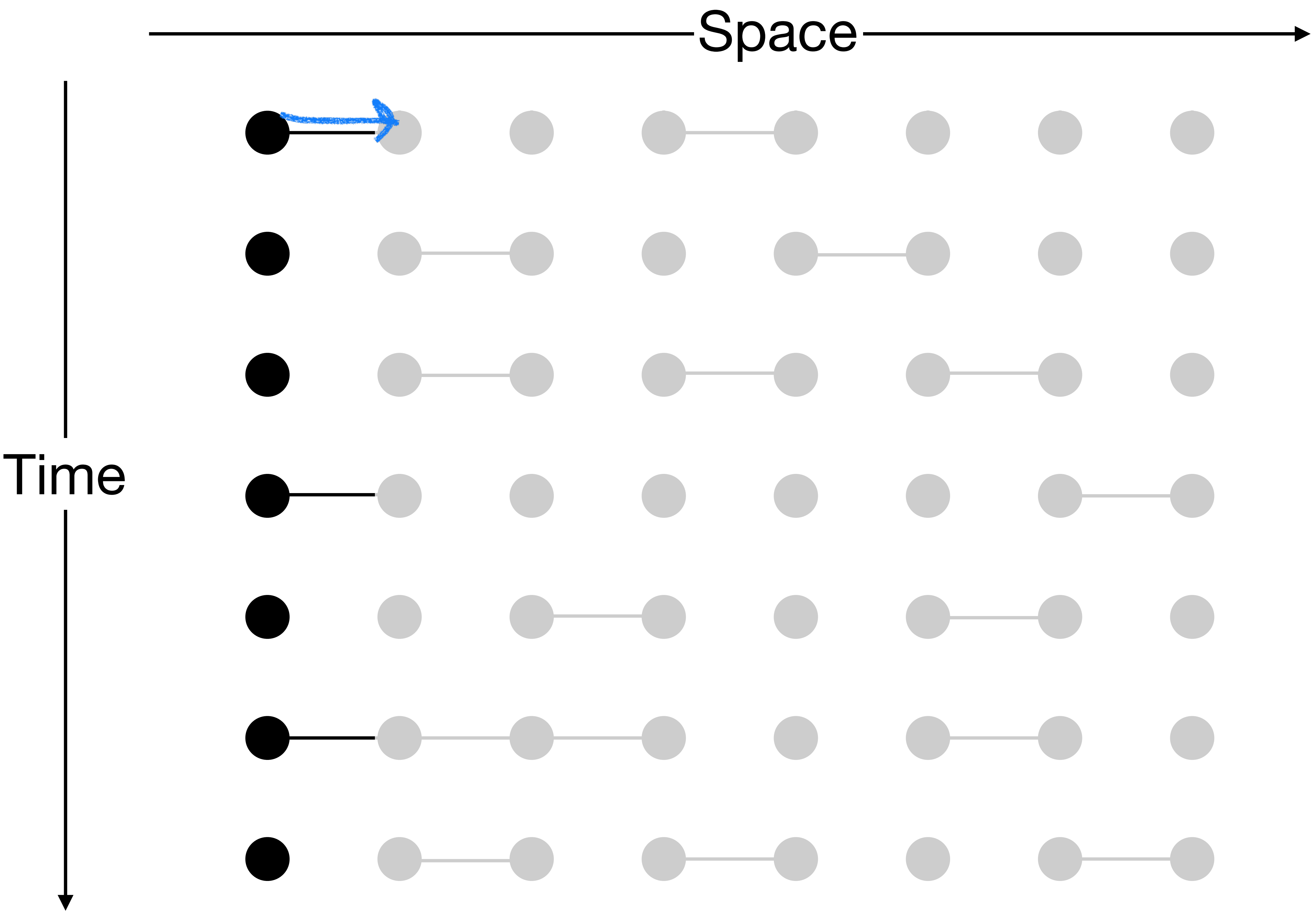
This is optimal

Time-Online Algorithm

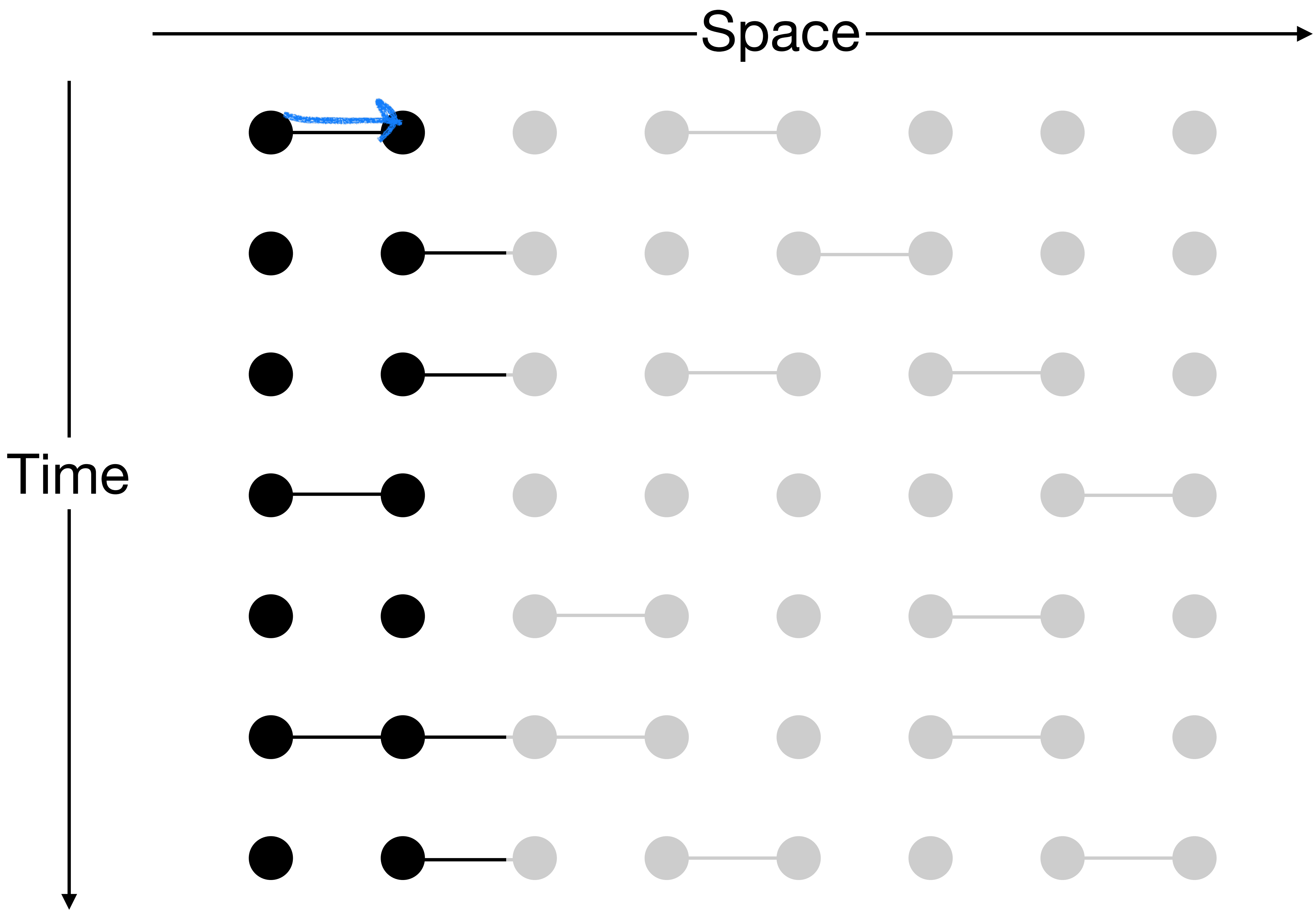
Let G_i be



Space-Online Algorithm



Space-Online Algorithm



Space-Online Algorithm Lower Bound

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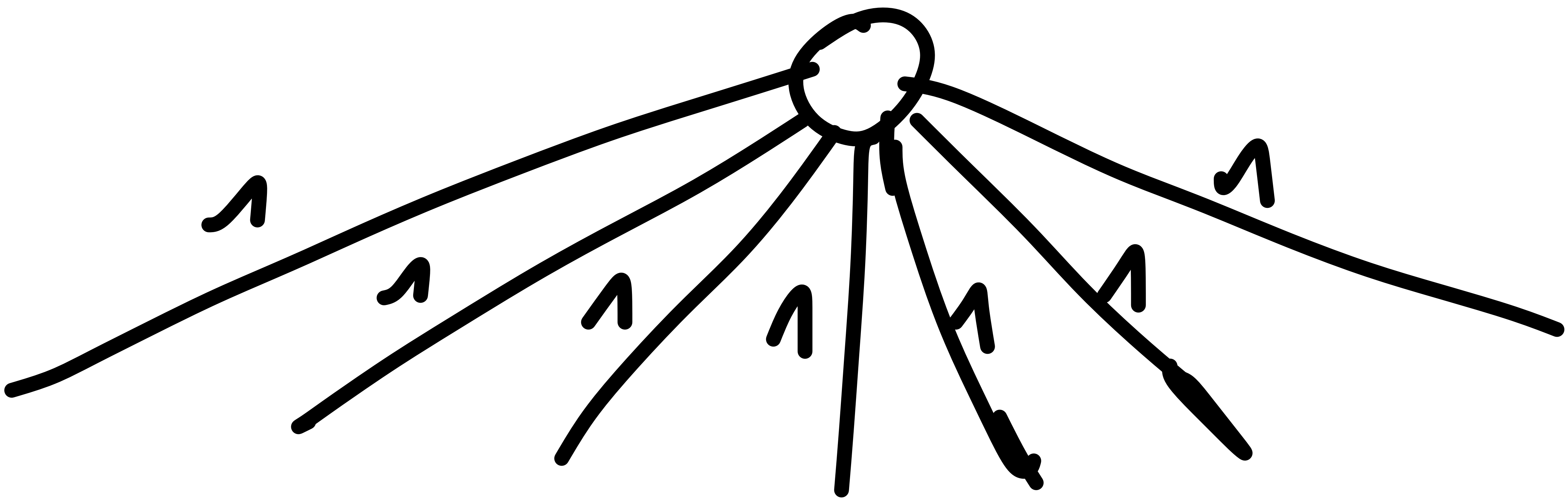

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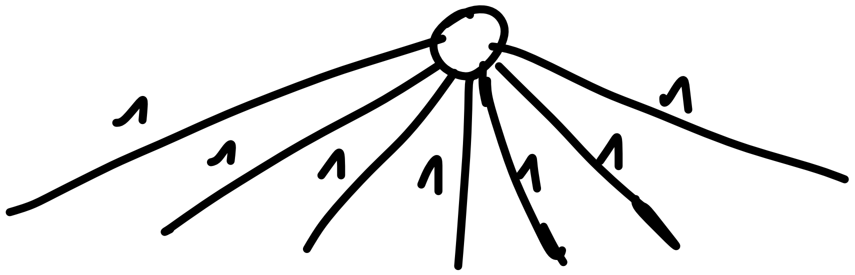

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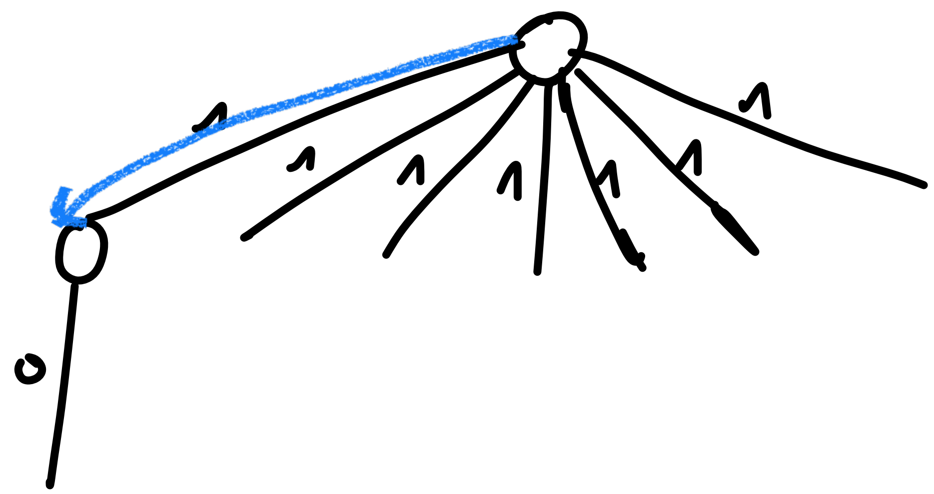

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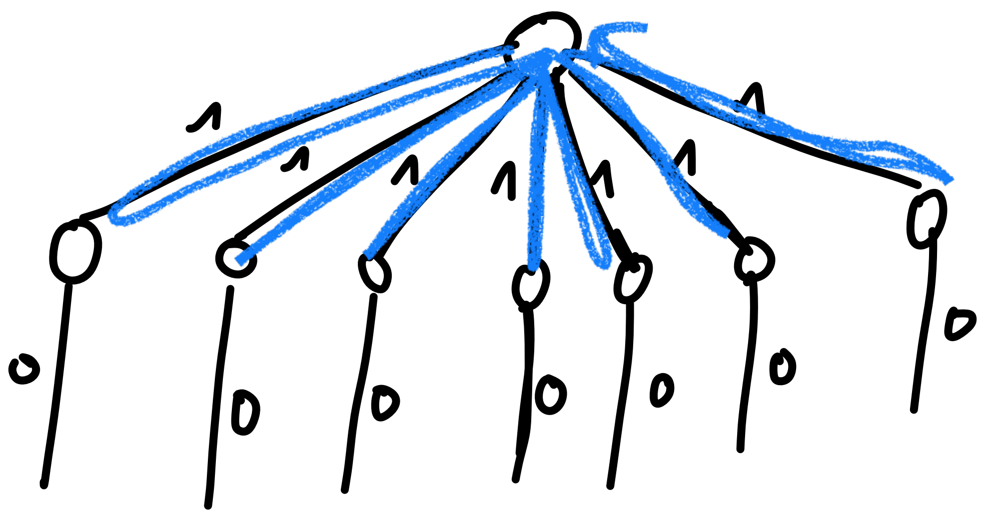

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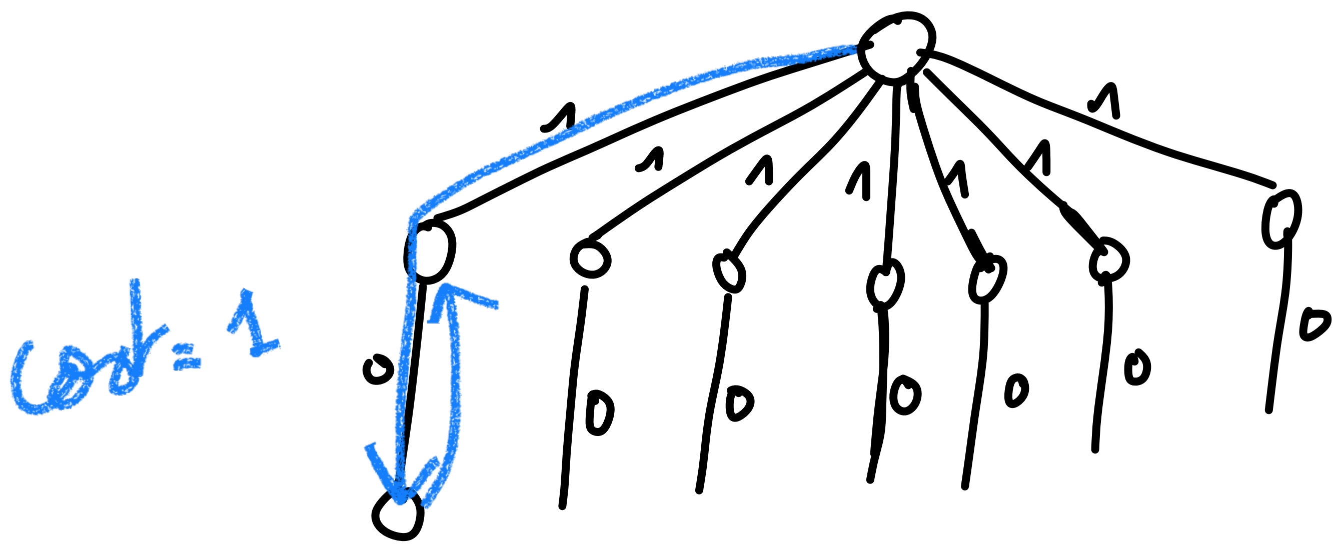

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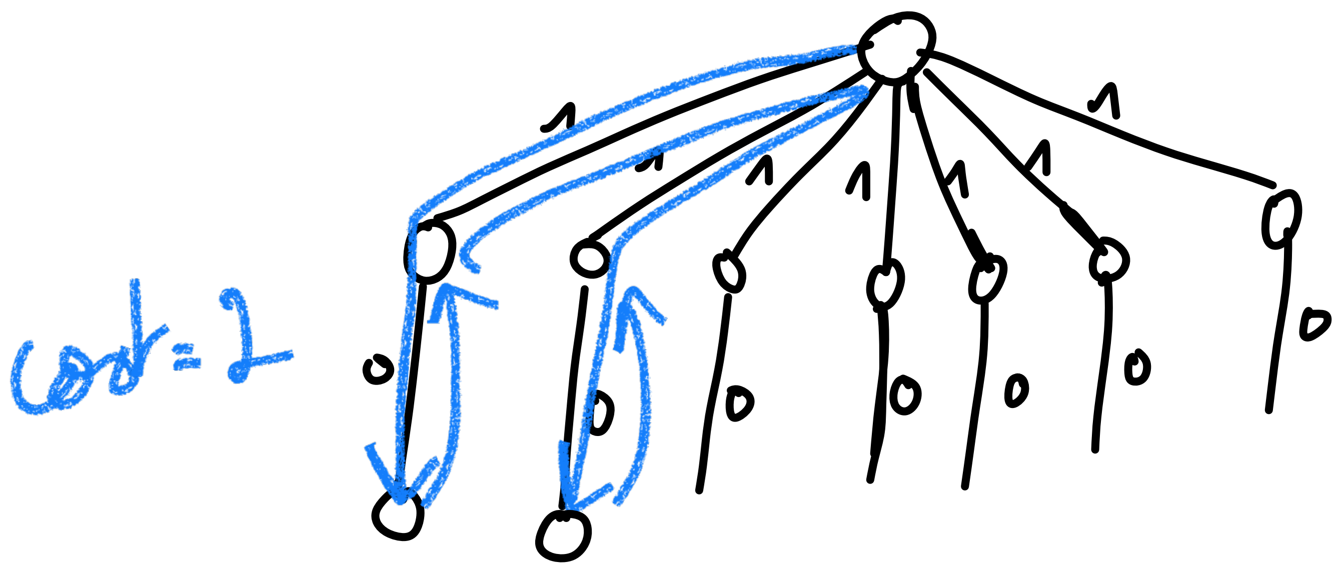

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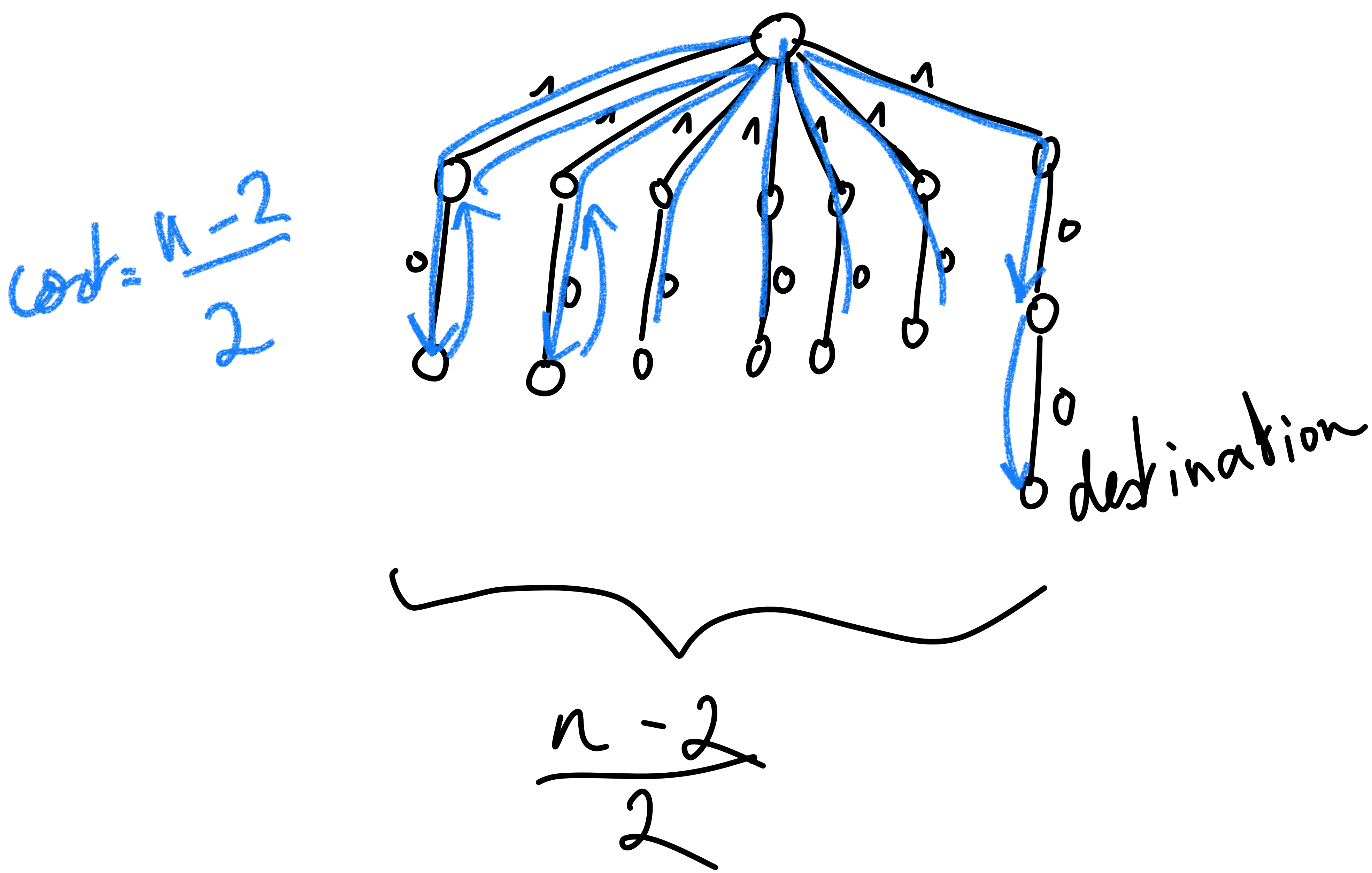

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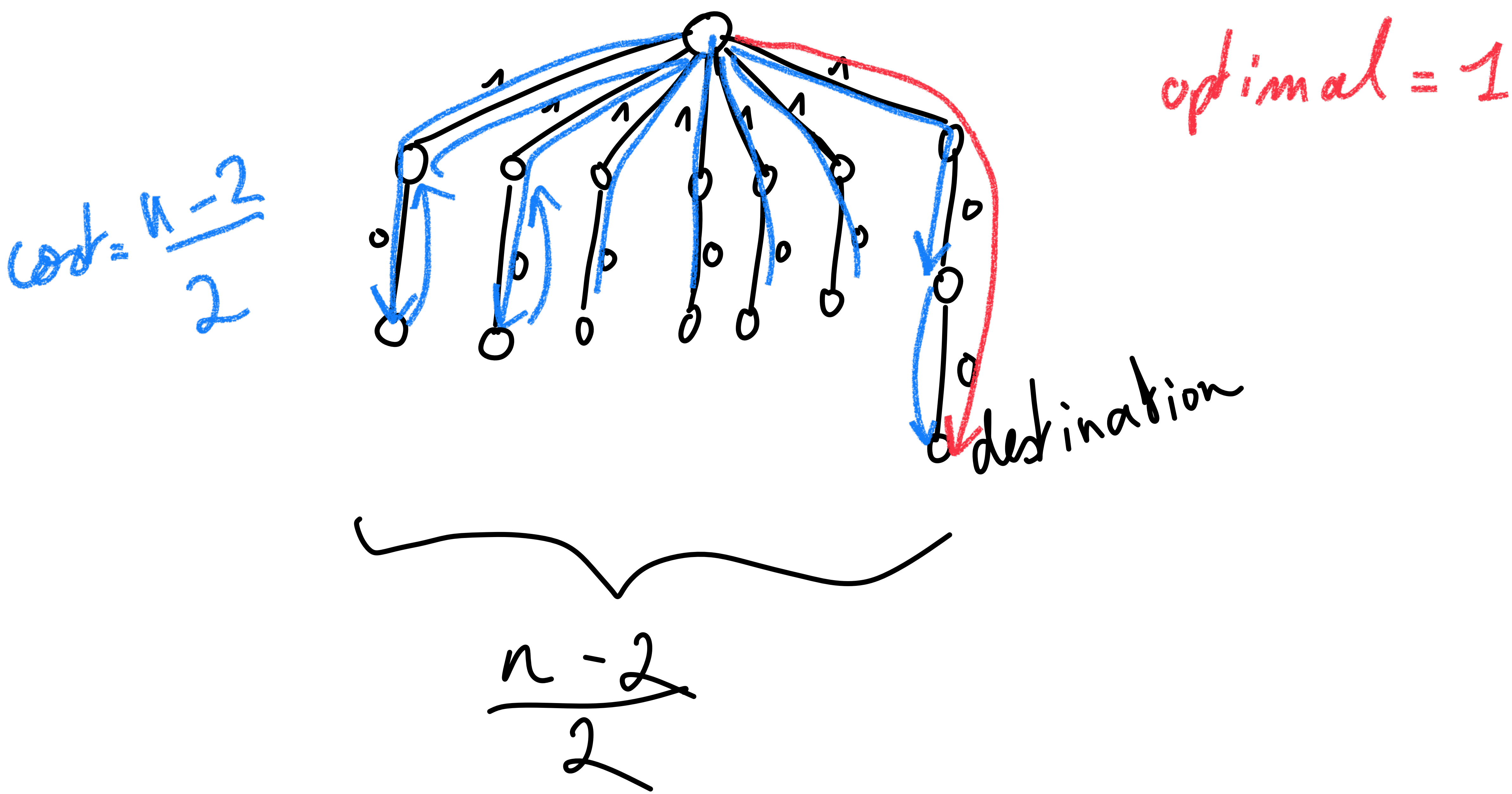

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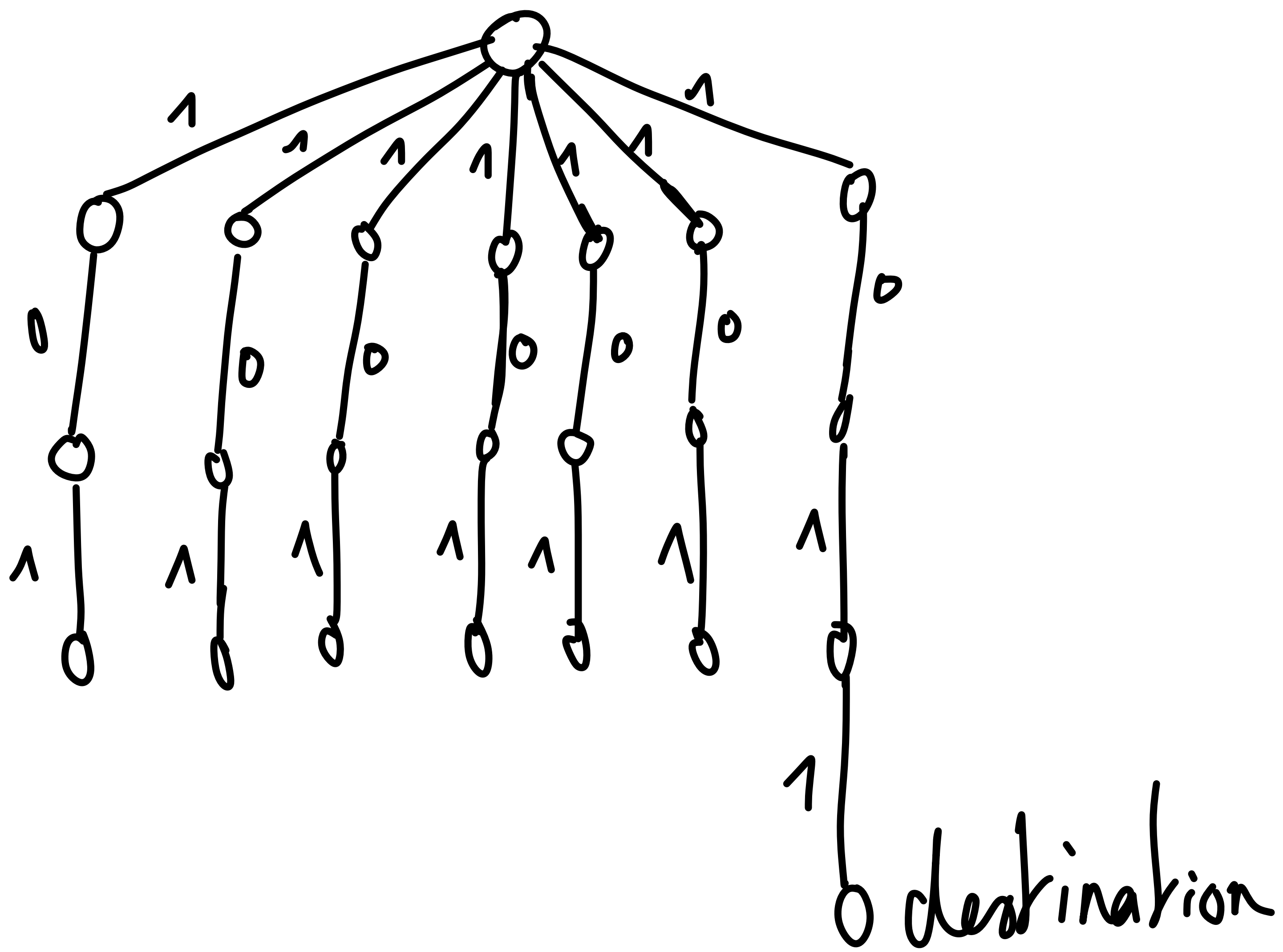

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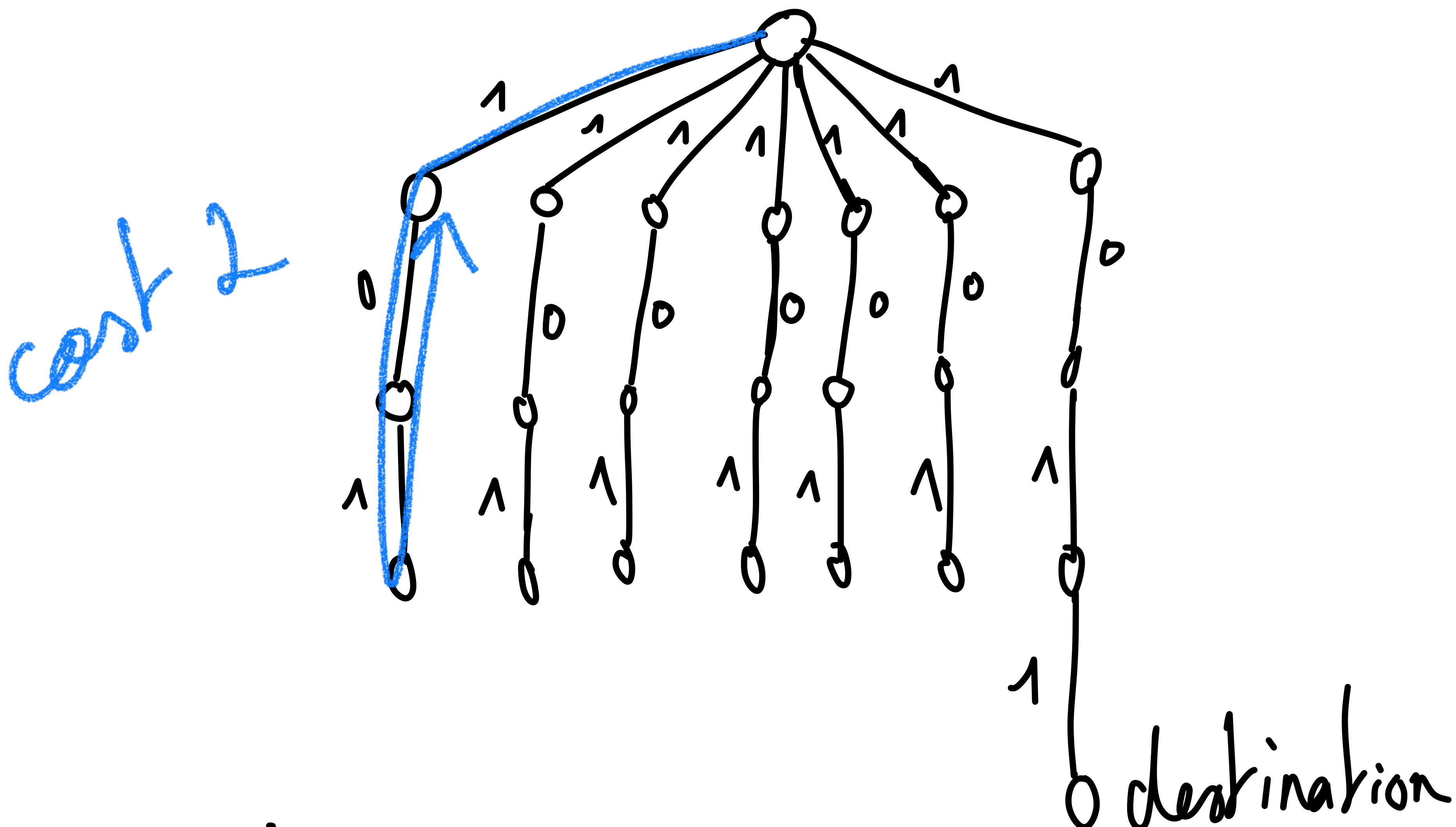
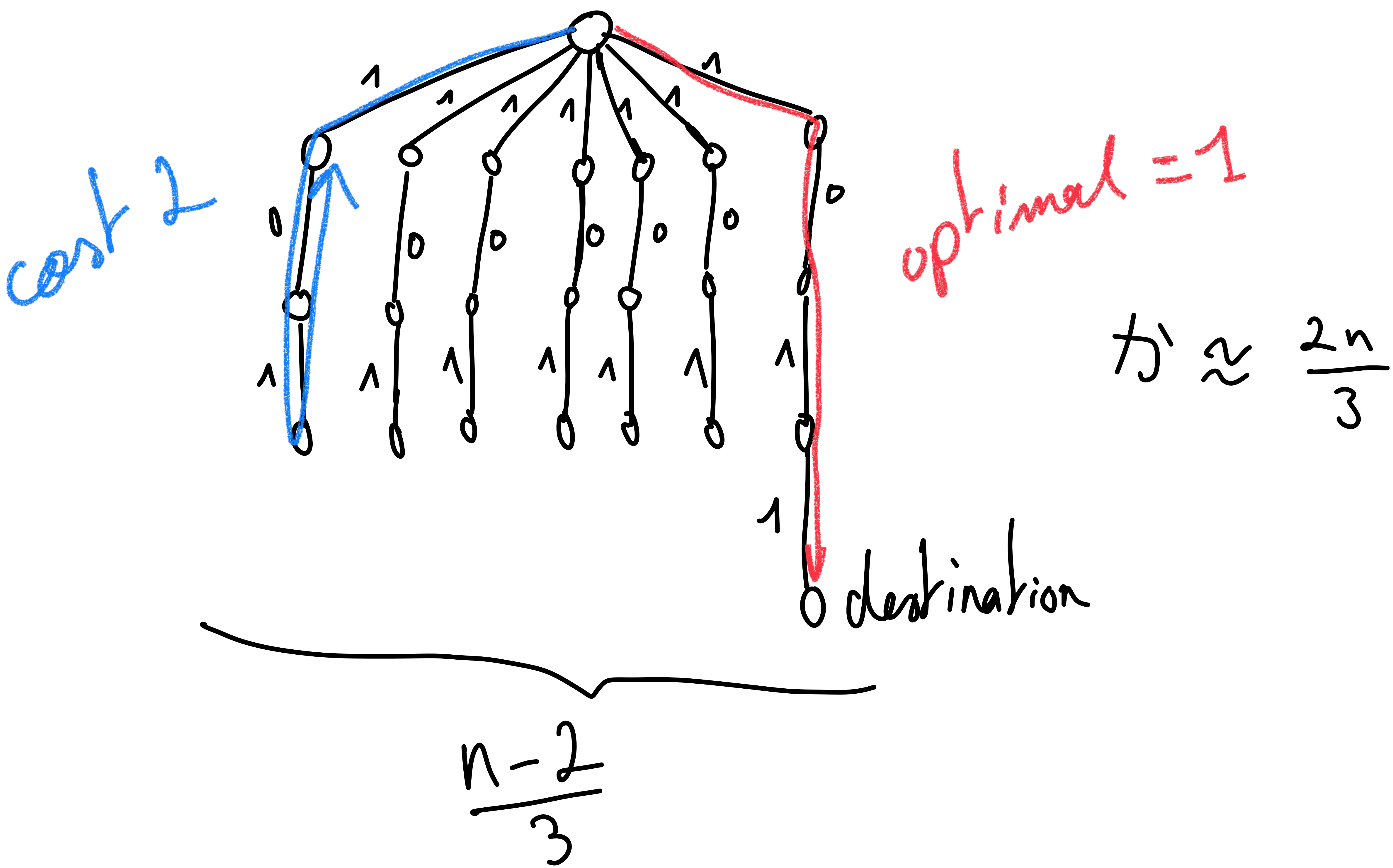


$\frac{n-2}{3}$

Space-Online Algorithm Lower Bound



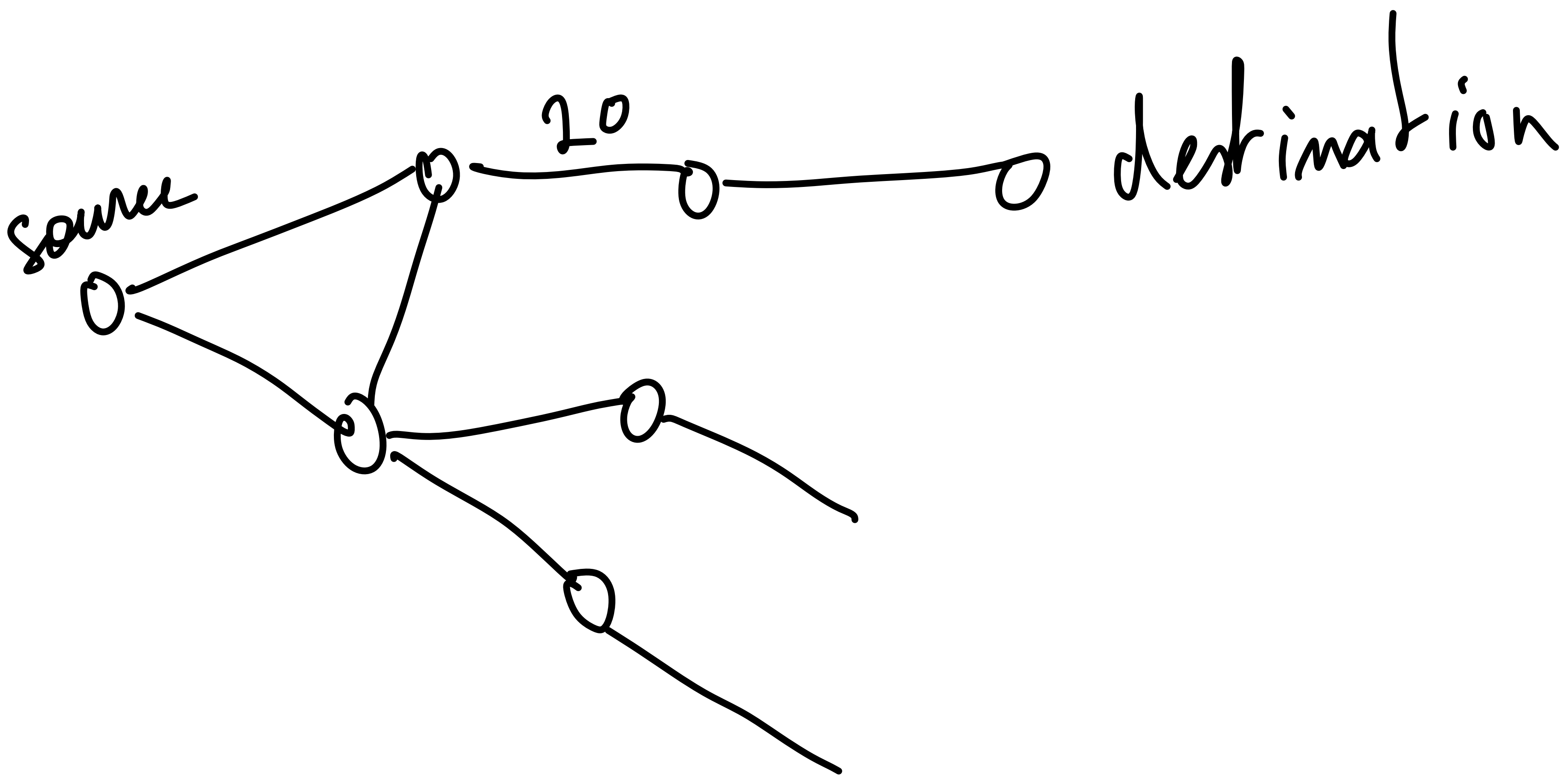
Space-Online Algorithm Lower Bound

No!

$\frac{2n}{3}$ is our best current LB

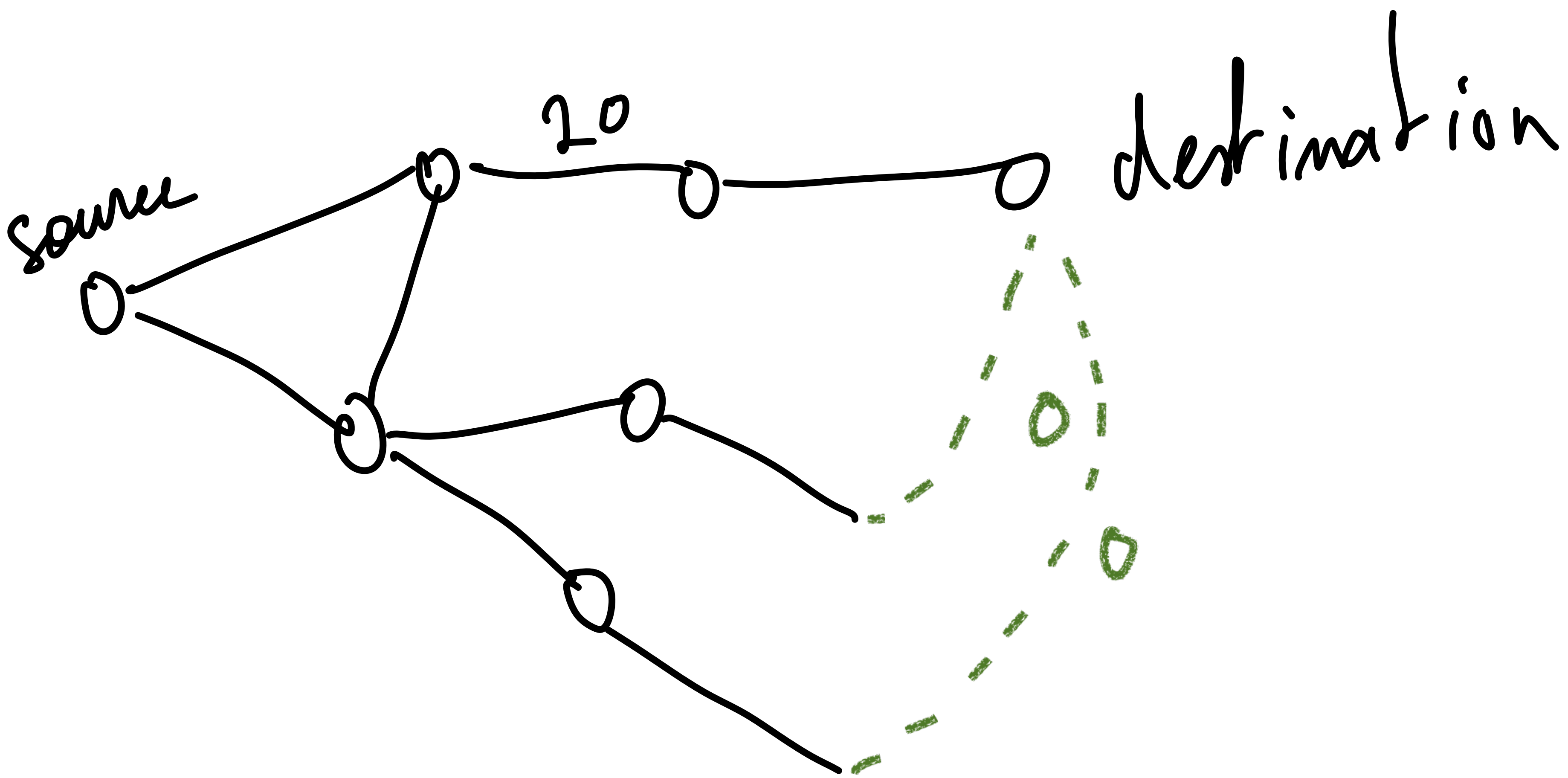
Our Space-Online Algorithm

Algo S-online



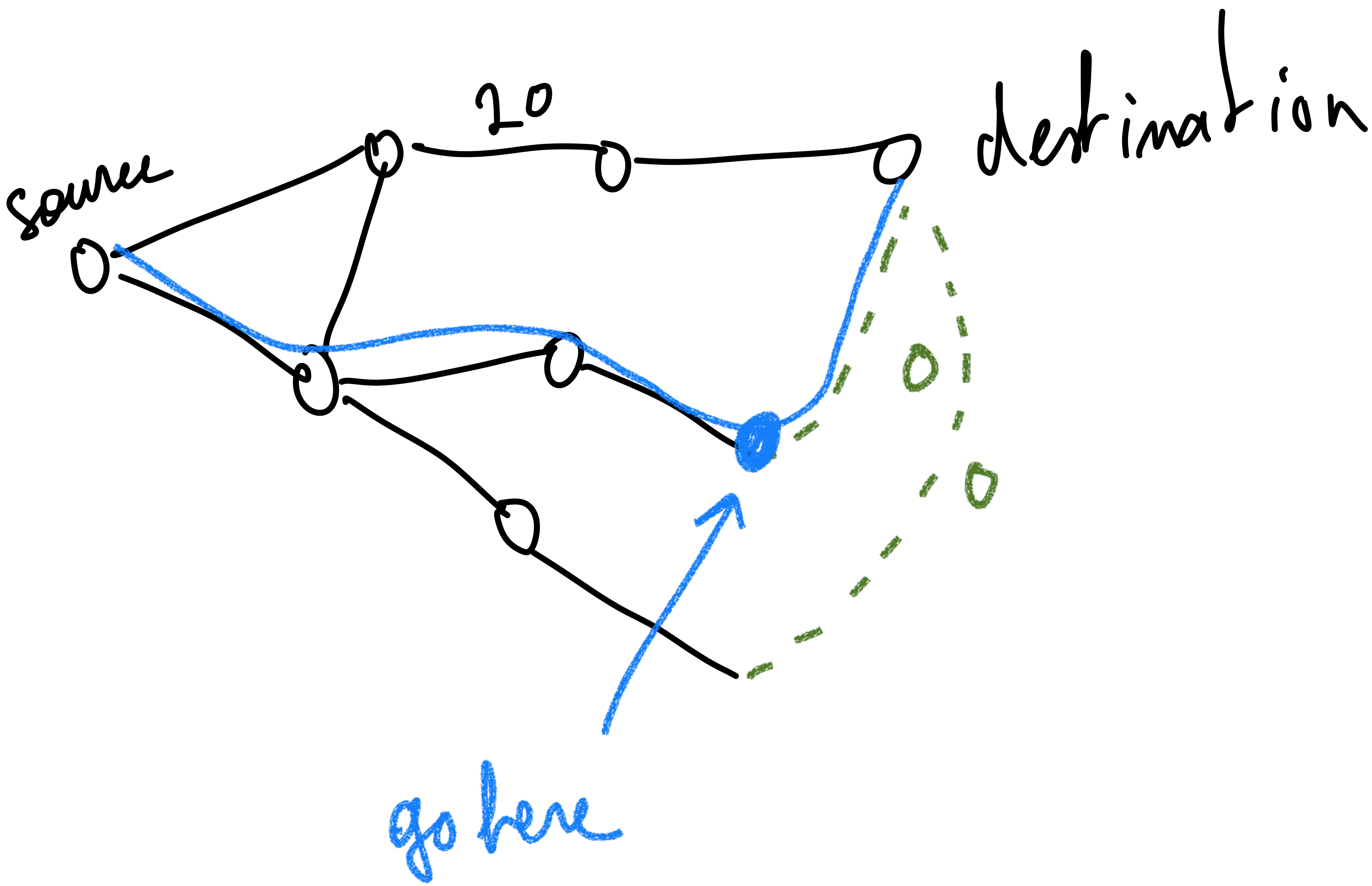
Our Space-Online Algorithm

Algo S-online



Our Space-Online Algorithm

Algo S-online



Our Space-Online Algorithm

Algo S-online

Our Space-Online Algorithm

Algo S-online

Each step cost $\leq$ optimal

Our Space-Online Algorithm

Algo S-online

* Each step cost $\leq$ optimal
* At most n-1 steps

Our Space-Online Algorithm

Algo S-online

* Each step cost $\leq$ optimal
* At most $n-1$ steps

力 $\leq n-1$

Conclusion

- Optimal Time-Online algorithm
- Upper and lower bound for the Space-Online problem

- The punchline comes before the question
- What’s the problem with Time Travel jokes ?

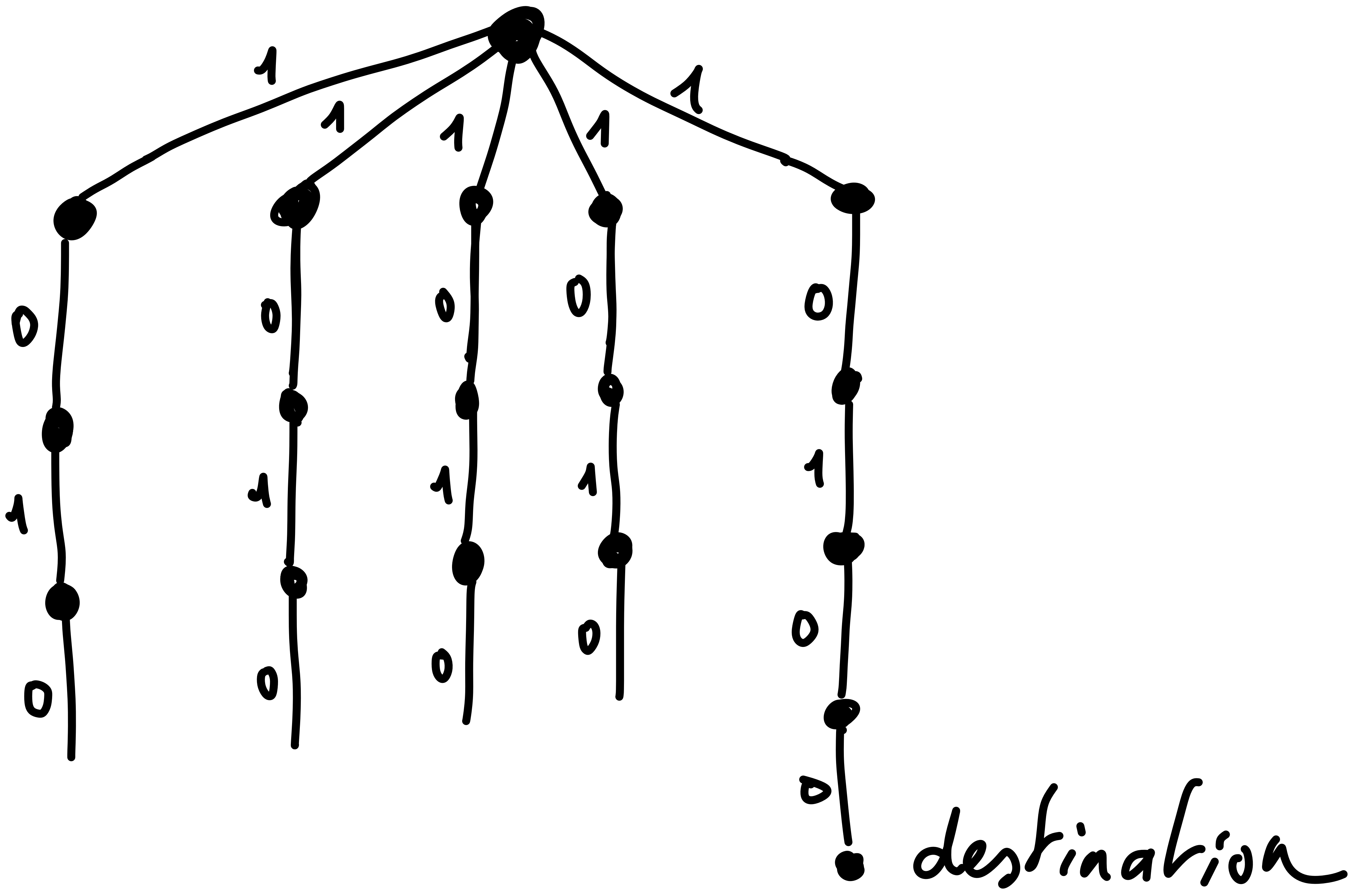
Conclusion

- Optimal Time-Online algorithm
- Upper and lower bound for the Space-Online problem

- The punchline comes before the question
- What’s the problem with Time Travel jokes ?

Thank you!

NO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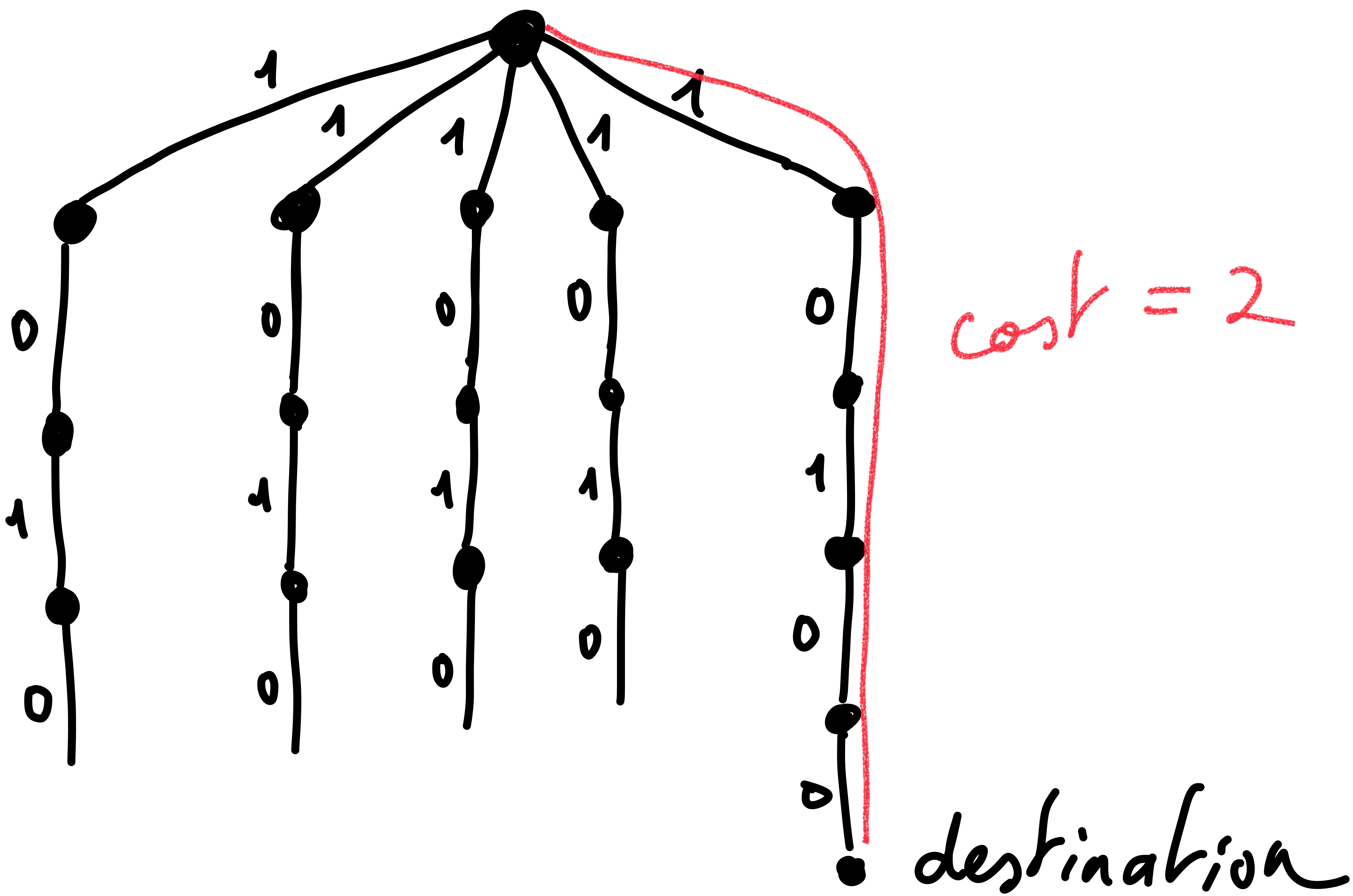
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cost = 2
destination