Crash-tolerant Exploration of Trees by Energy Sharing Mobile Agents

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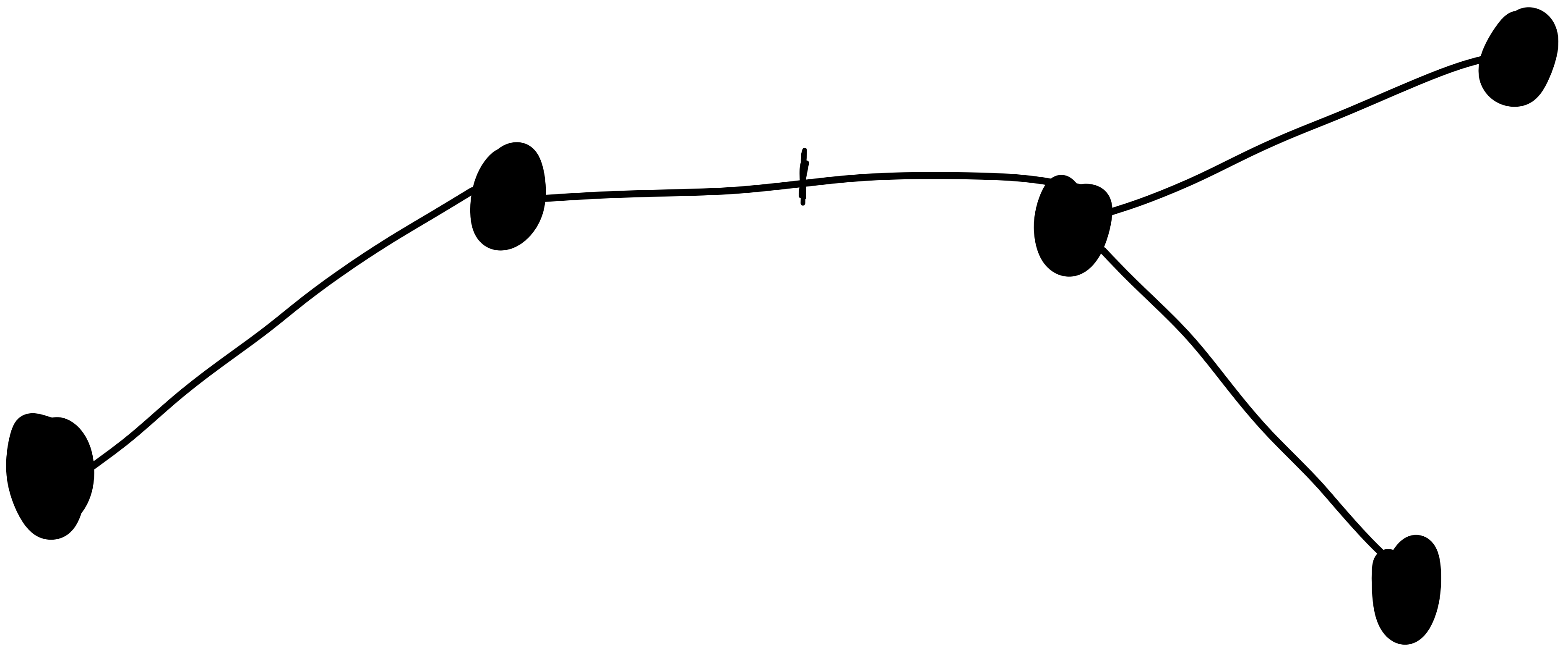
Mobile agents with energy



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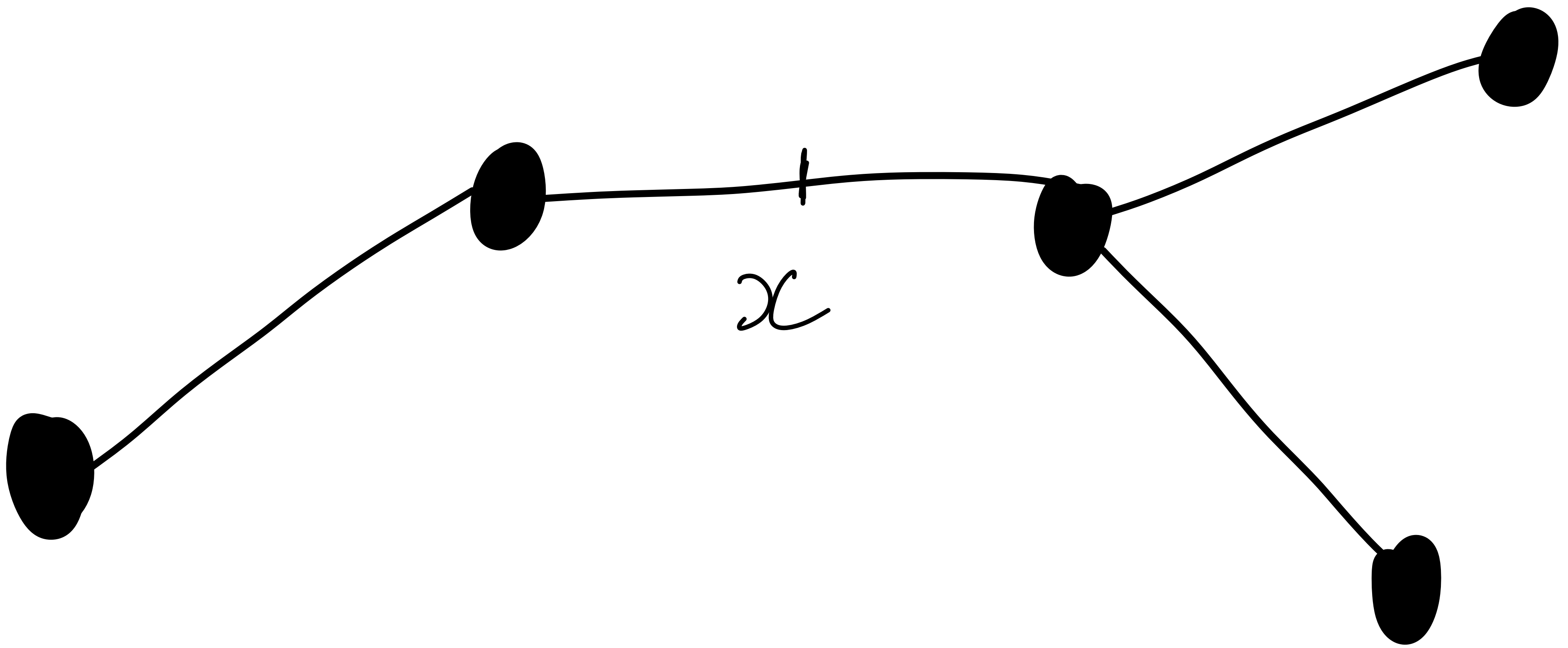

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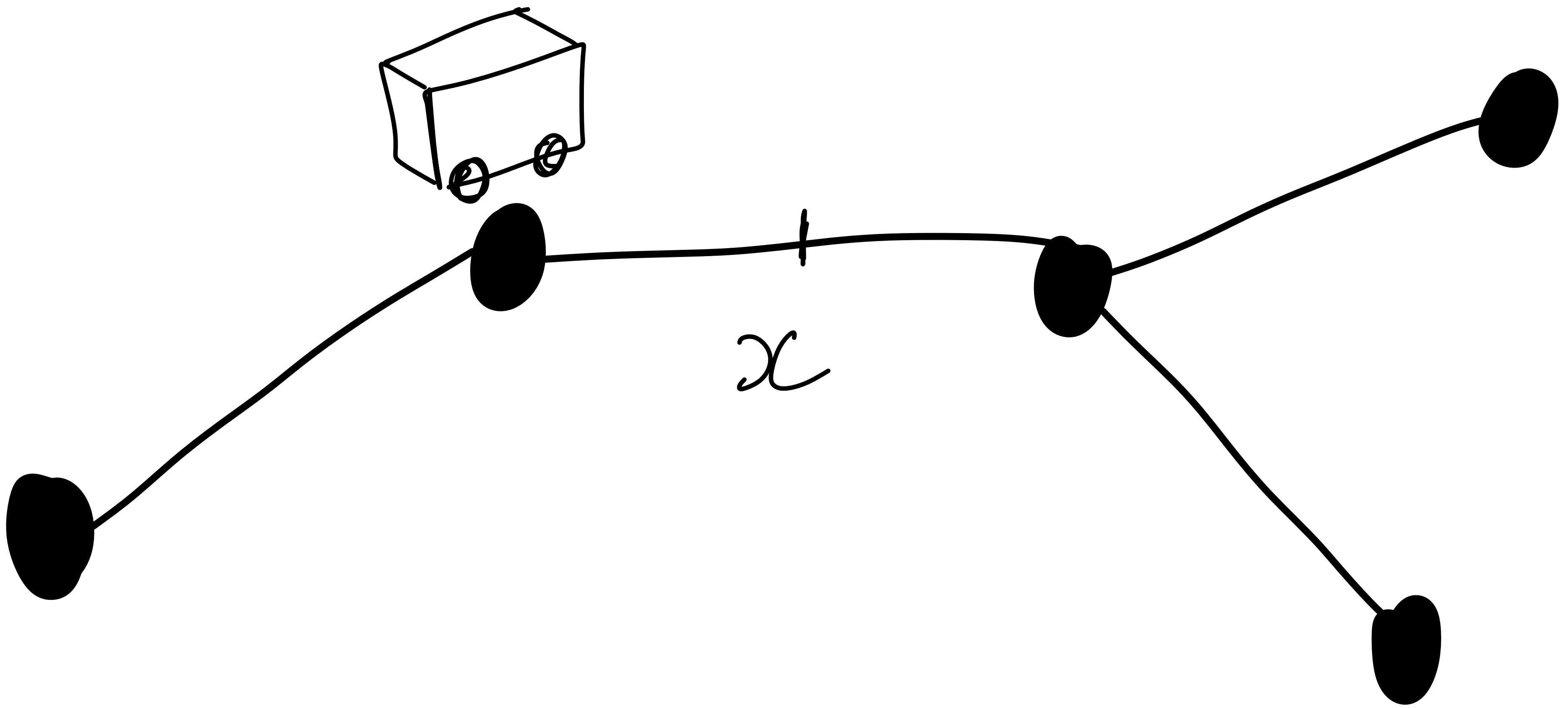

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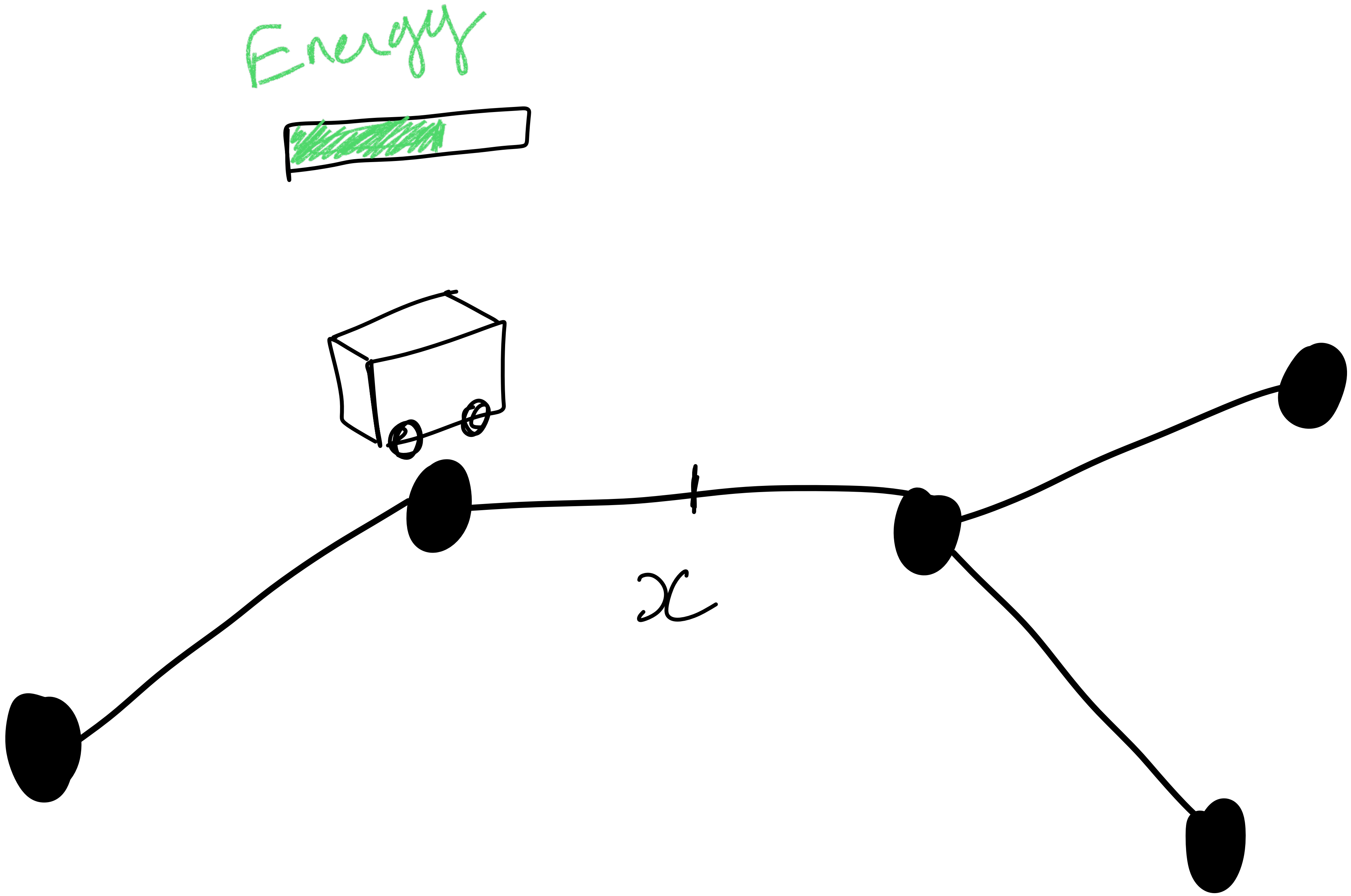

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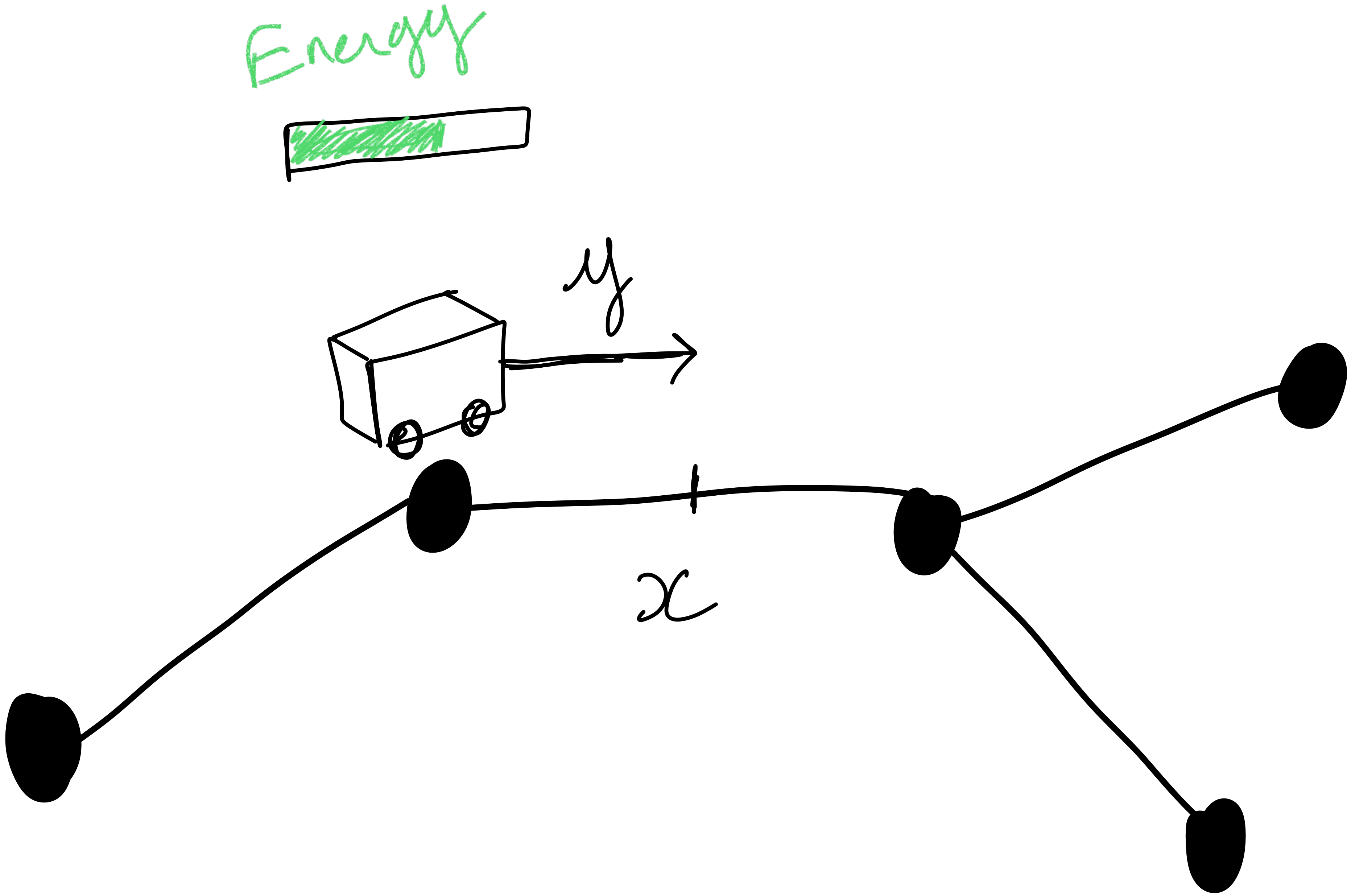

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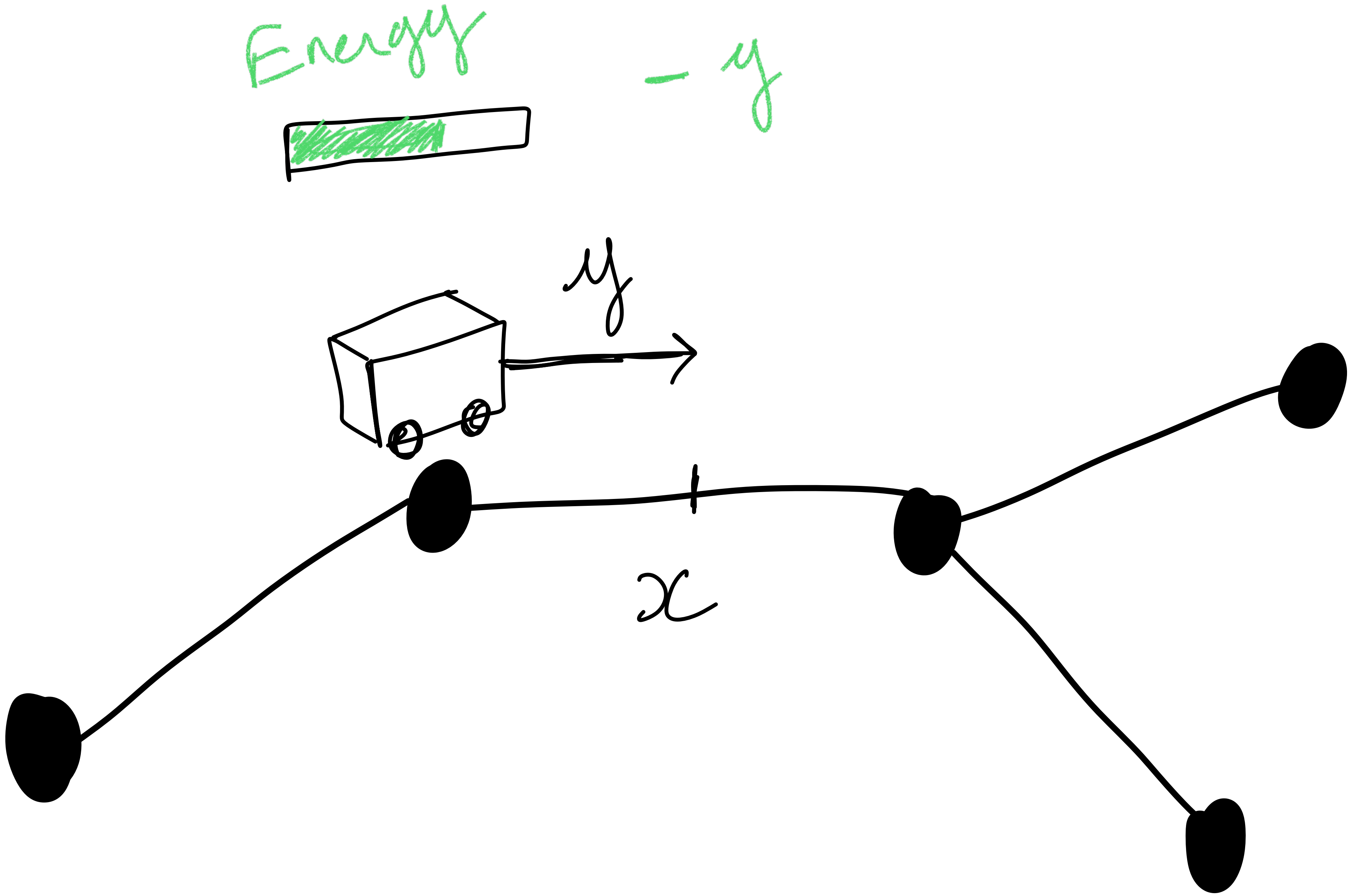
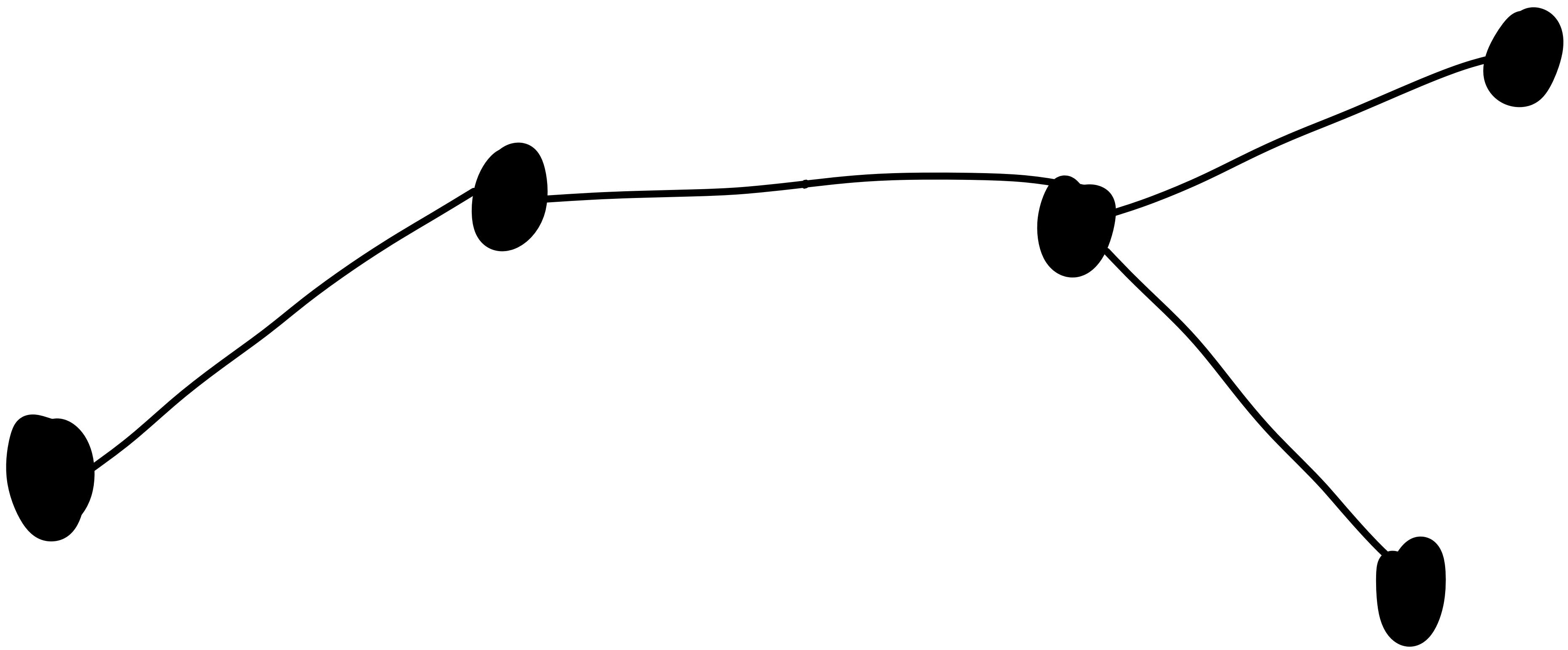


Graph exploration



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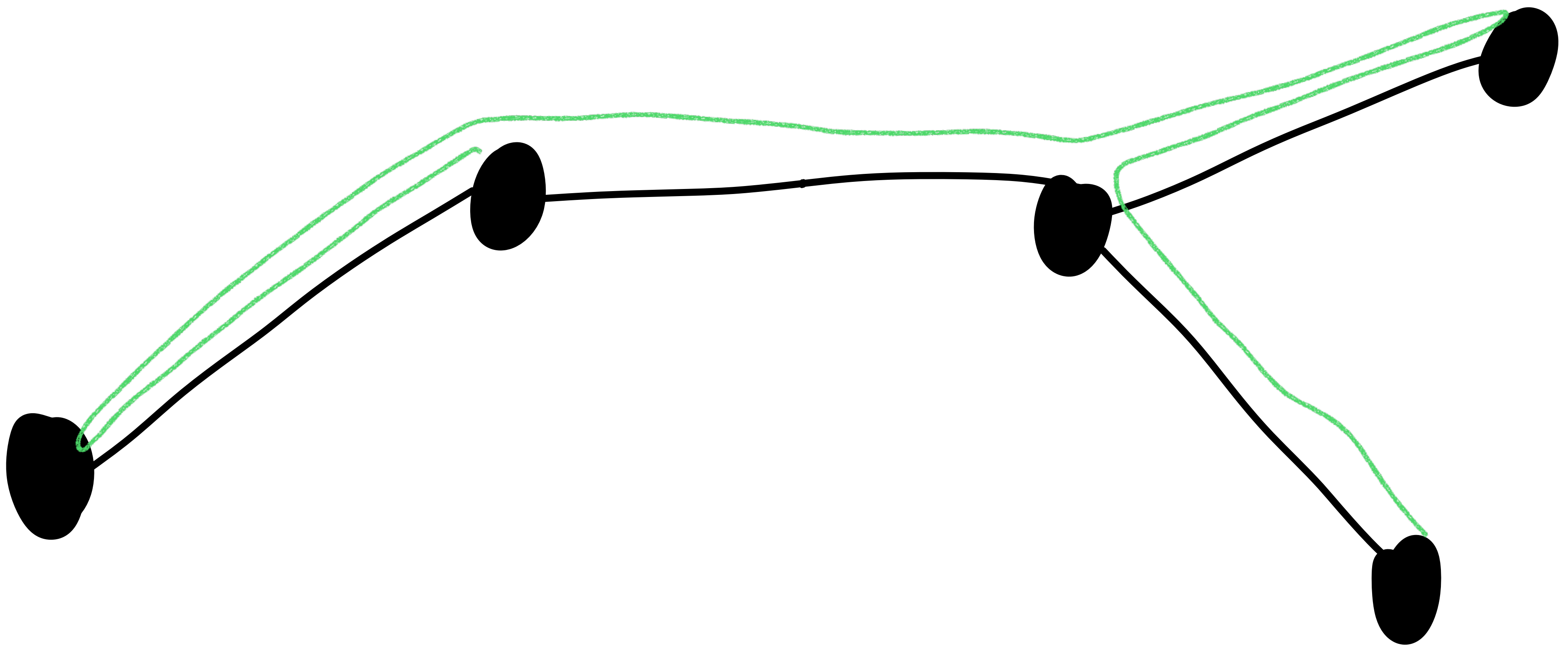

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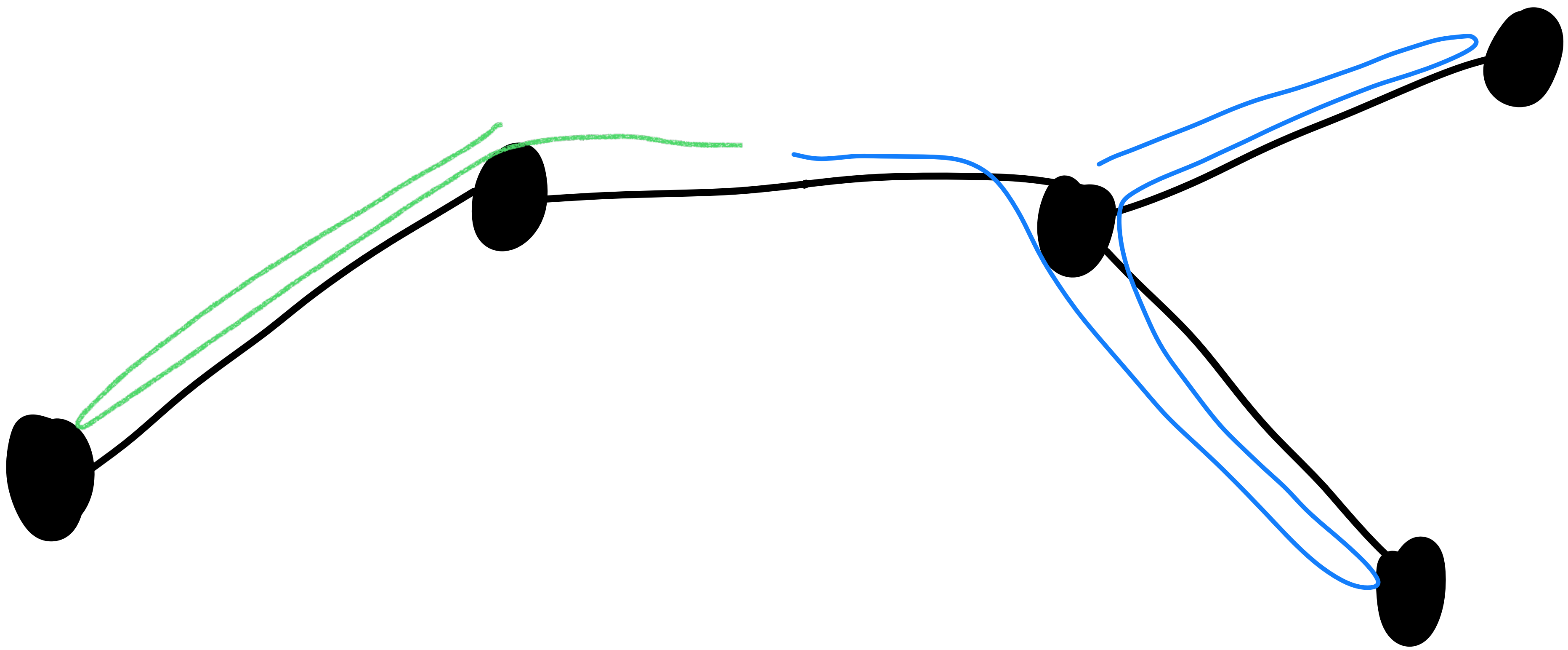

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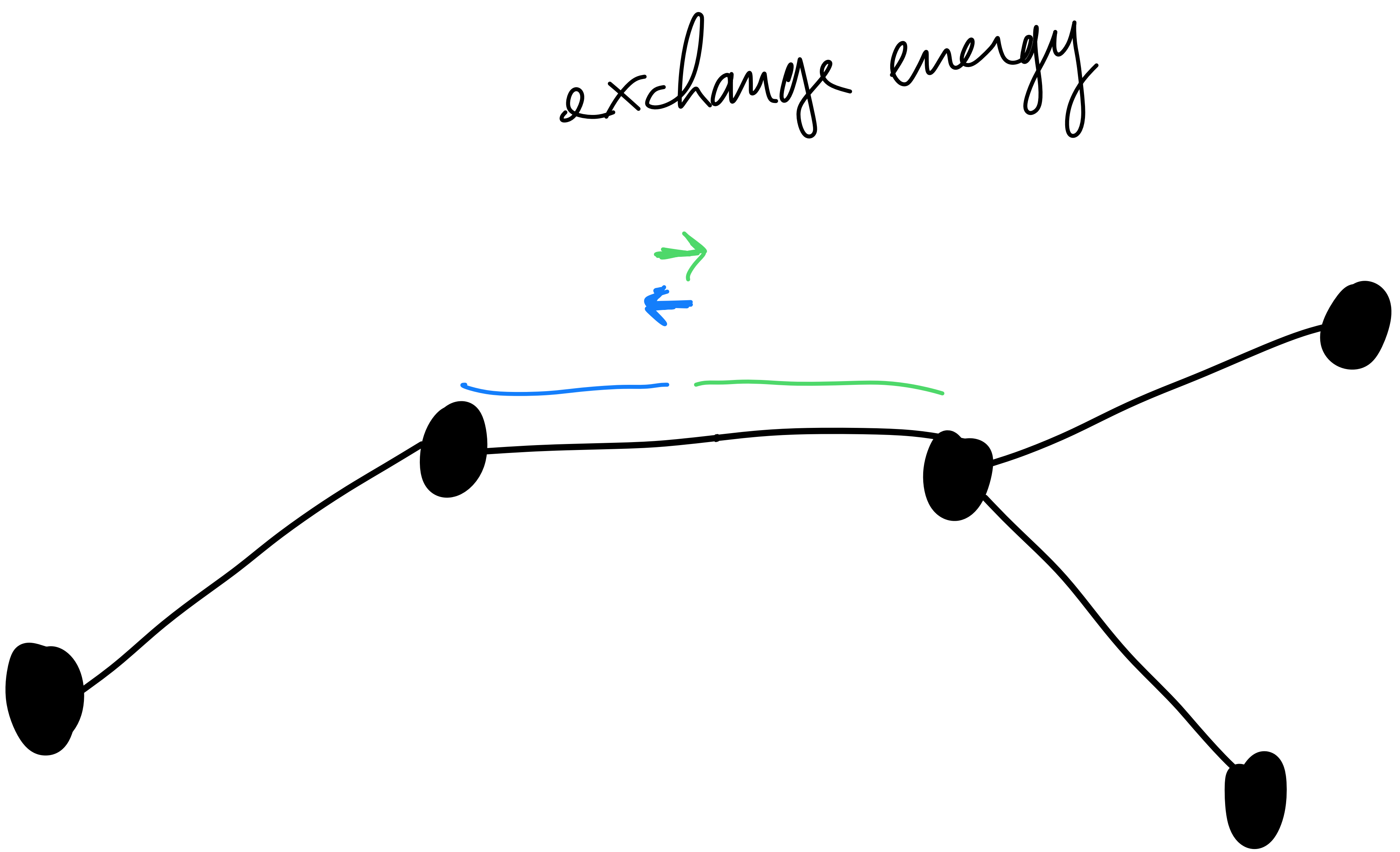

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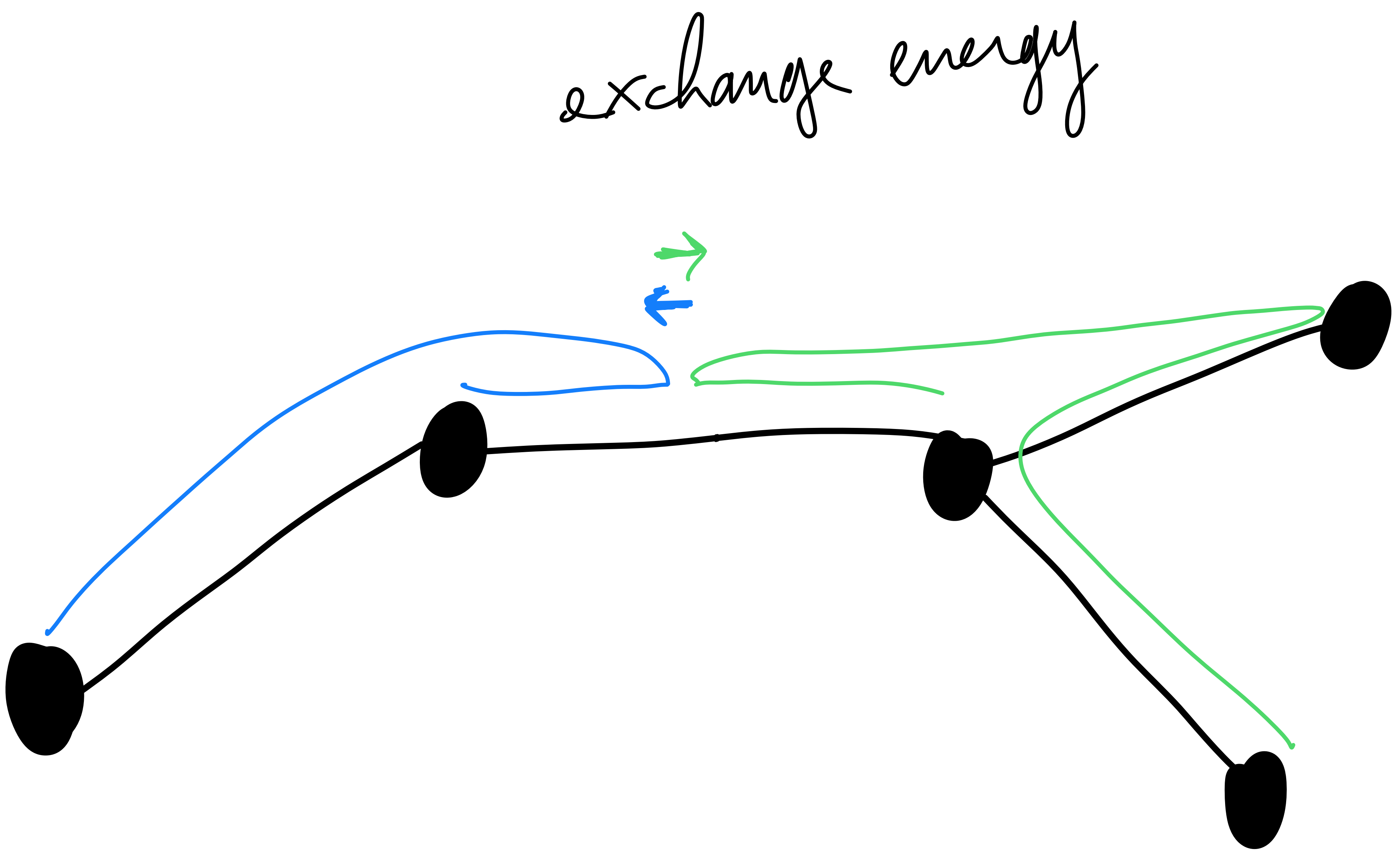

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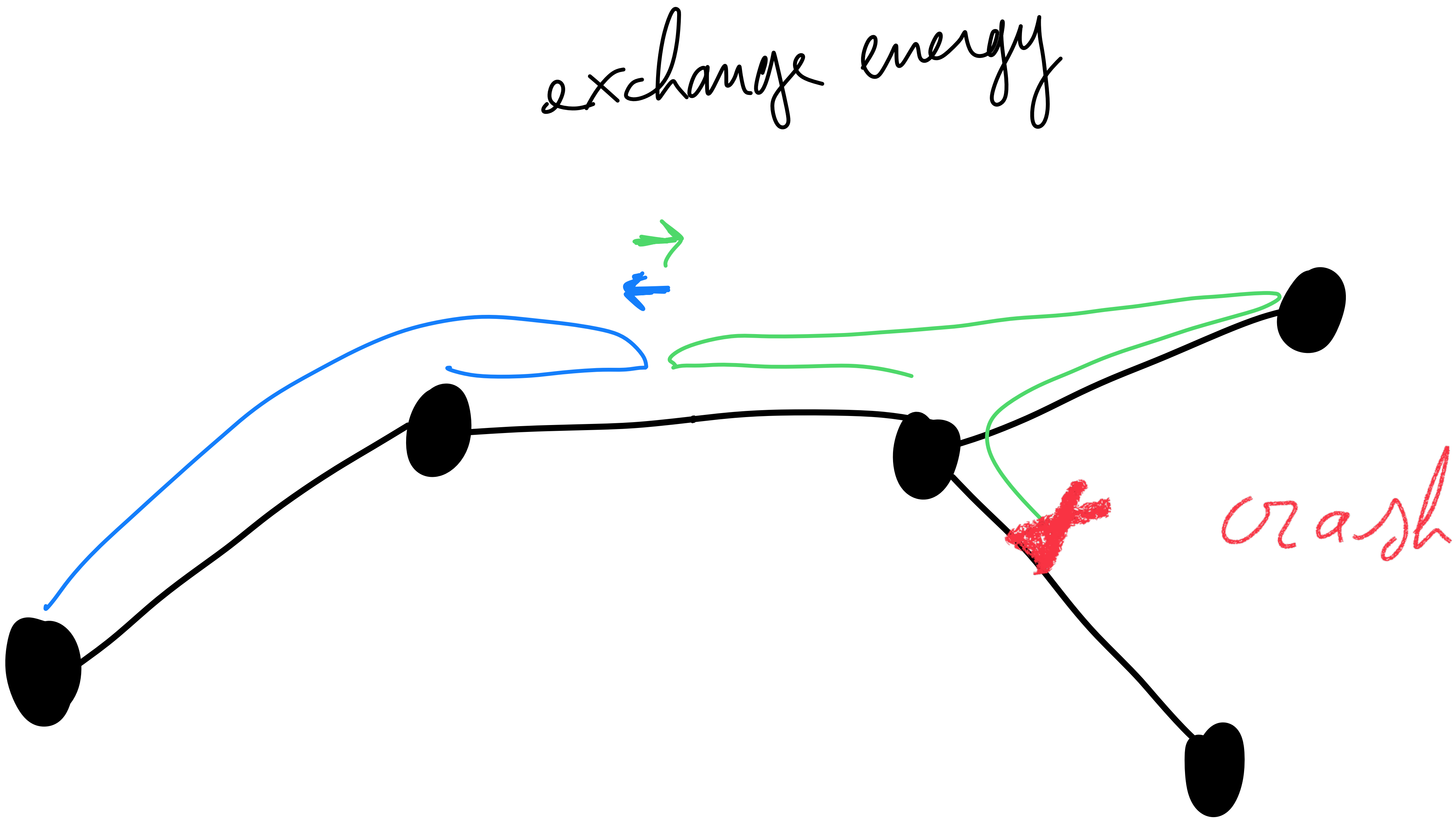

Localized algorithms

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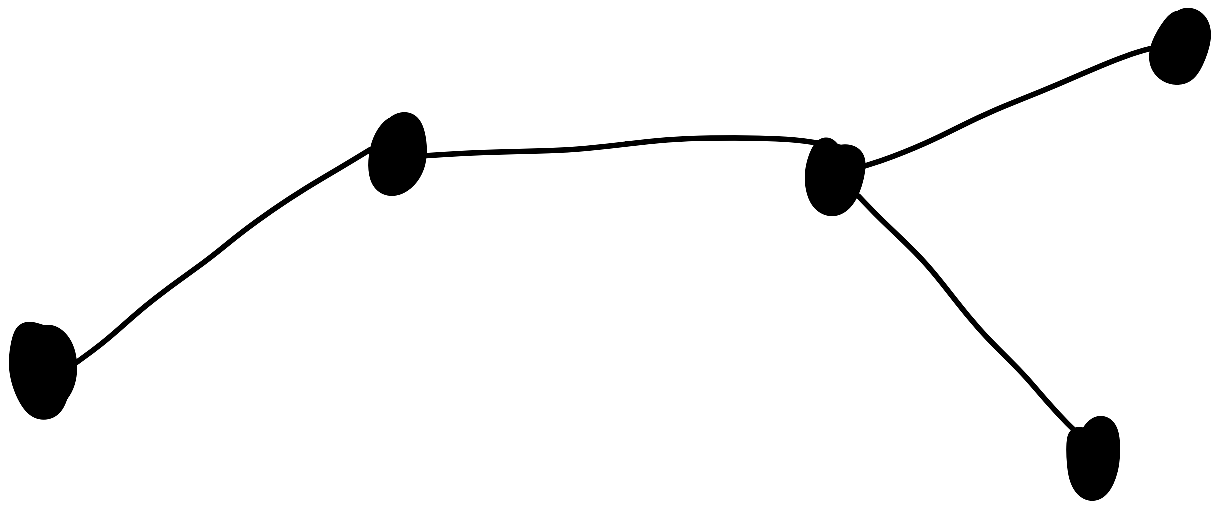
Given:



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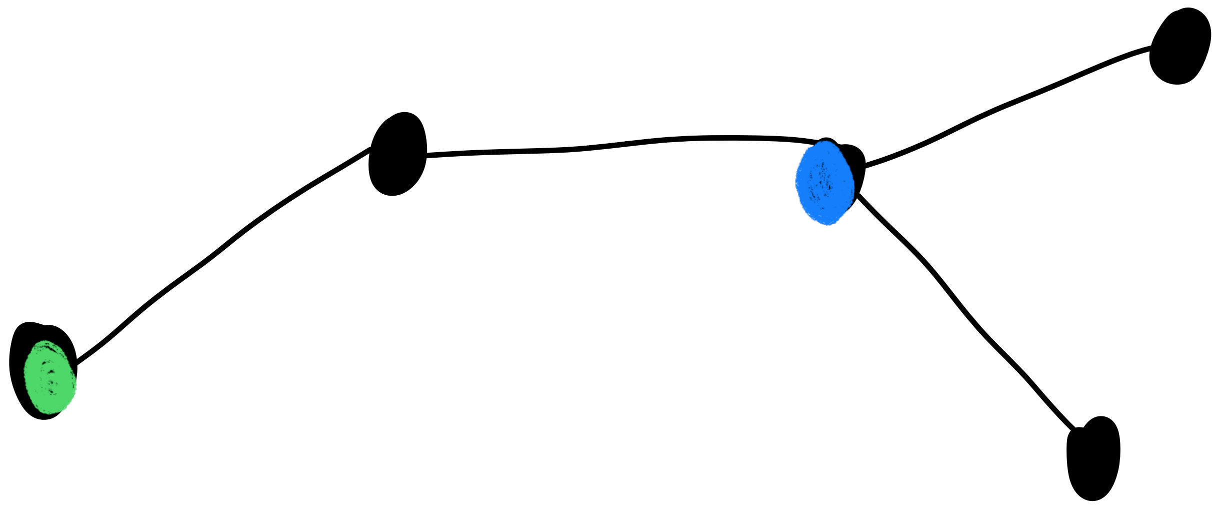

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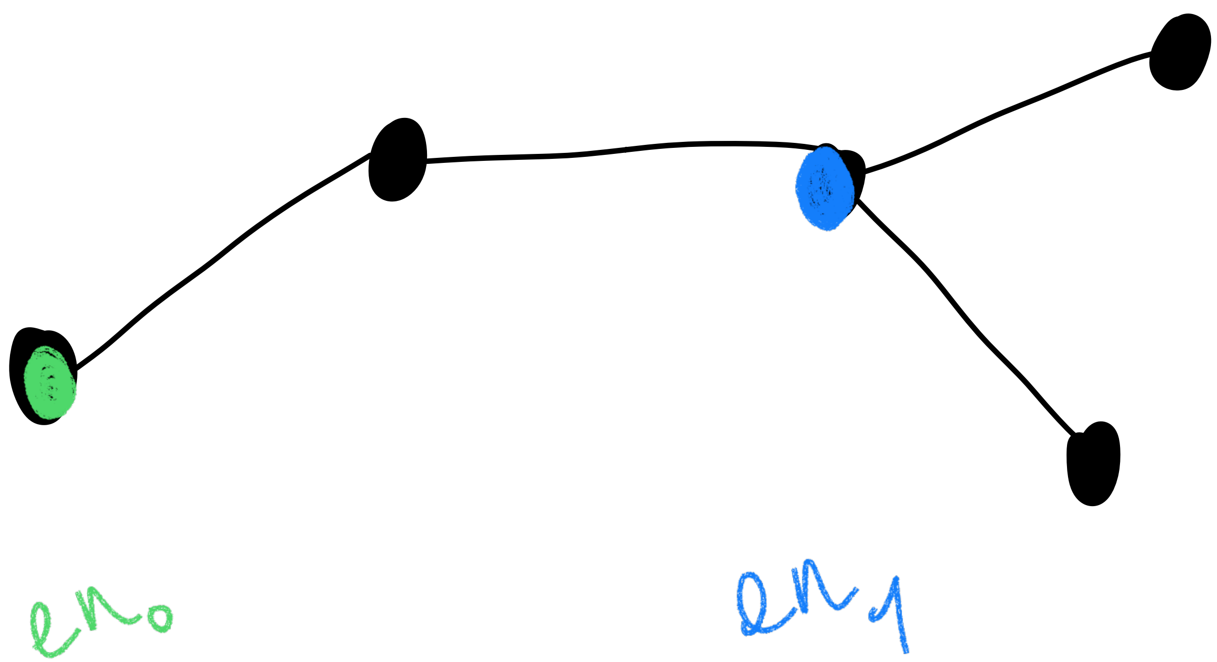

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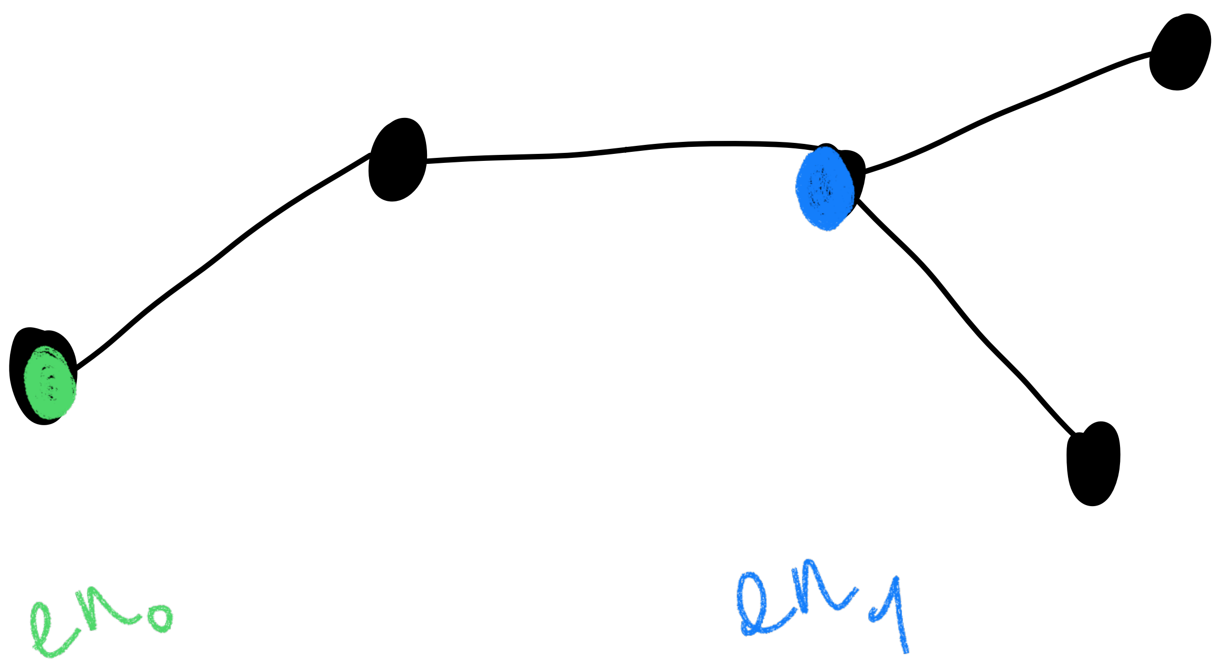
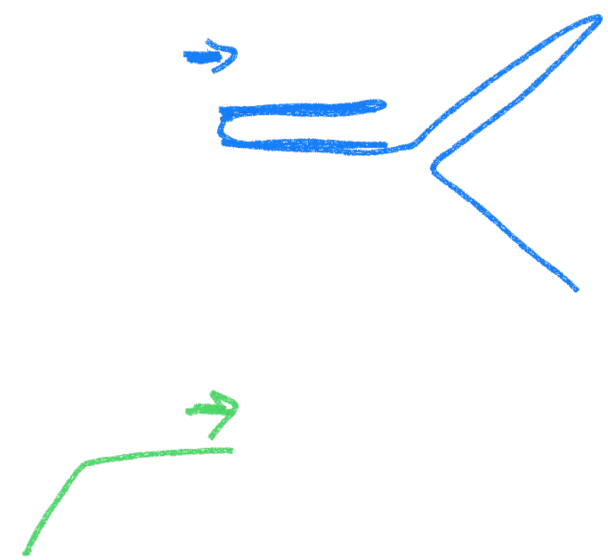


Find:

2 Algorithms



Localized algorithms

Find:

2 Algorithms

$\Rightarrow$

Once deployed,

- autonomous
- communicate when meet

Synchronous

or

Asynchronous

Exploring the line with 2 agents

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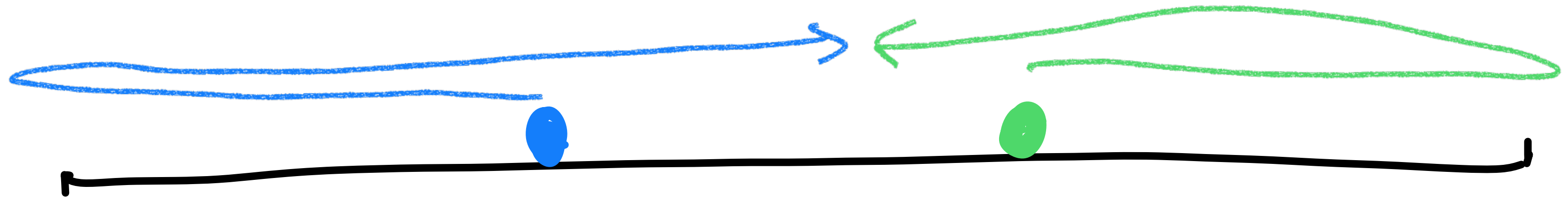

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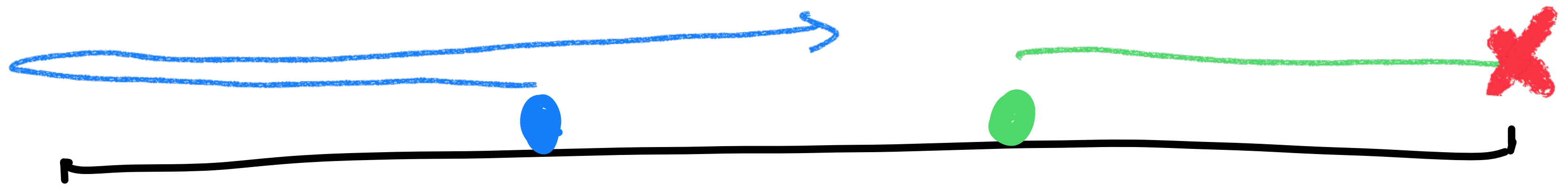
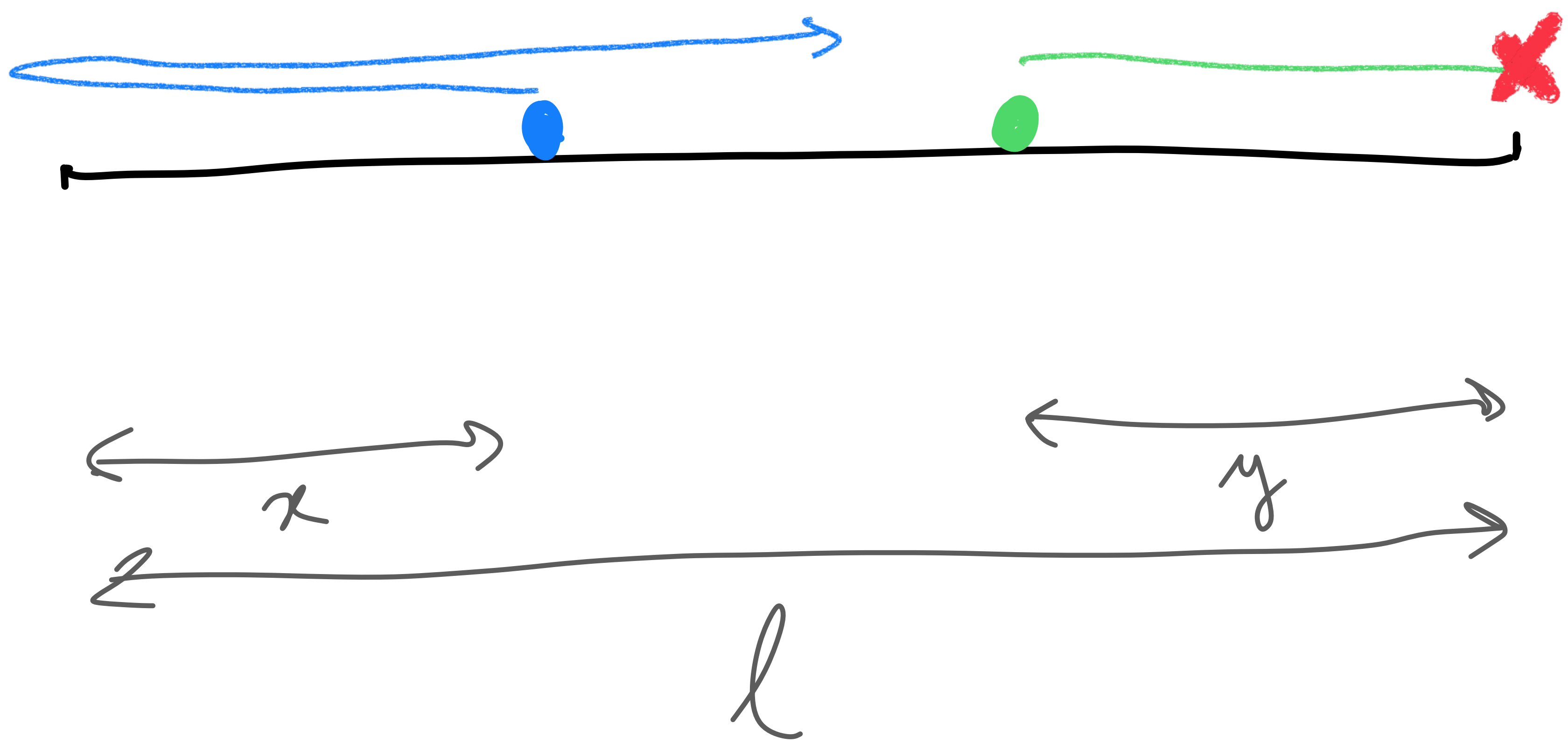


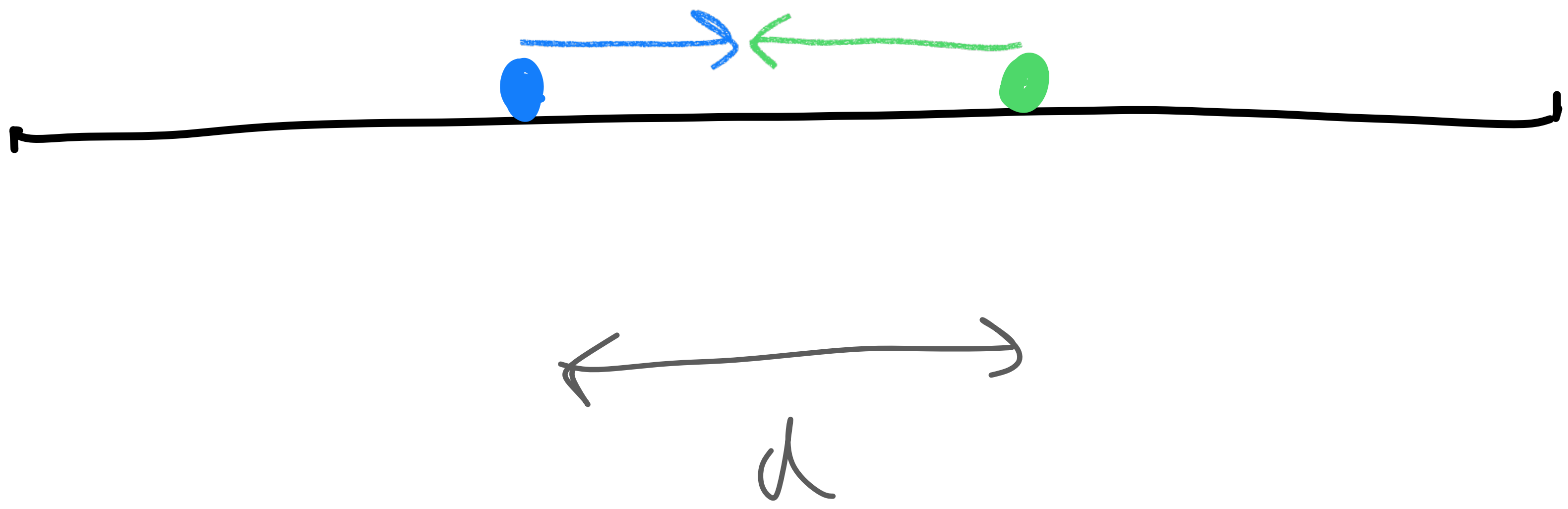
One easy algorithm

$$en_0 \geq \ell + x \ \wedge \ en_1 \geq \ell + y$$



Another easy algorithm

$$en_0 \geq d \ \wedge \ en_1 \geq d \ \wedge \ en_0 + en_1 \geq 3\ell + d$$



How many easy algorithms are there?

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We found the necessary conditions and the corresponding algorithms

$$\boldsymbol{c_1} : (en_0 \geq x + y) \wedge (en_1 \geq y) \wedge (en_0 + en_1 \geq 2\ell + x + y)$$
$$\boldsymbol{c_2} : (en_0 \geq \ell - x) \wedge (en_1 \geq 2\ell - (x + y)) \wedge (en_0 + en_1 \geq 4\ell - (x + y))$$
$$\boldsymbol{c_3} : (en_0 \geq \ell + x) \wedge (en_1 \geq 2\ell - y)$$
$$\boldsymbol{c_4} : (en_0 \geq y - x) \wedge (en_1 \geq y - x) \wedge (en_0 + en_1 \geq min(3\ell + y - x, 2\ell - x + 3y))$$

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Same for the synchronous case

And for trees?

Almost tight algorithms

Thank you

Synchronous lines

Action a_1

r_0 : $(v_0) \leftarrow; \rightarrow (v_{n-1})$

r_1 :

wait $x + y$ *time units for* r_0 **then**

$\rightarrow (v_{n-1})$

timeout

$(r_0) \leftarrow$;

if $\overleftarrow{r_0}$ **then**

$(v_0) \leftarrow$; $\rightarrow (v_{n-1})$

else $\rightarrow (v_{n-1})$;

end

Action a_2

r_0 :

wait $2\ell - (x + y)$ *time units for* r_1

then

$(v_0) \leftarrow$

timeout

$\rightarrow (r_1)$;

if $\overrightarrow{r_1}$ **then**

$\rightarrow (v_{n-1})$; $(v_0) \leftarrow$

else $(v_0) \leftarrow$;

end

r_1 : $\rightarrow (v_{n-1}); (v_0) \leftarrow$

Action a_3

r_0 : $(v_0) \leftarrow; \rightarrow (v_{n-1})$;

r_1 : $\rightarrow (v_{n-1}); (v_0) \leftarrow$

Action a_4

r_0 :

wait $y - x$ *time units for* r_1 **then**

wait $2x$ *time units for* r_1 **then**

$\rightarrow (v_{n-1})$

timeout

$(r_1) \leftarrow$;

if $\overrightarrow{r_1}$ **then** $\rightarrow (v_{n-1})$

else $(v_0) \leftarrow$; $\rightarrow (v_{n-1})$

end

timeout

$\rightarrow (r_1)$;

if $v_0 \prec v_{n-1}$ **then**

$(v_0) \leftarrow$; $\rightarrow (v_{n-1})$;

else $\rightarrow (v_{n-1})$; $(v_0) \leftarrow$;

end

r_1 : $(v_0) \leftarrow; \rightarrow (v_{n-1})$

Trees

$ct_1 \; : (en_0 \geq x) \wedge (en_1 \geq x) \wedge (en_0 + en_1 \geq 2W + 2d\lceil \log_{3/2} W \rceil + x + 2)$

The following shows the actions of the agents. It is assumed that subtrees $T_1, T_2, \ldots T_k$ ($k \leq \lceil \log_{3/2} W \rceil$) are a priori determined by the recursive centroid-based partitions.

Step 0: The agents meet on the shortest path between their initial locations. (This is executed only once at the beginning of the execution.) Set $i = 1$.

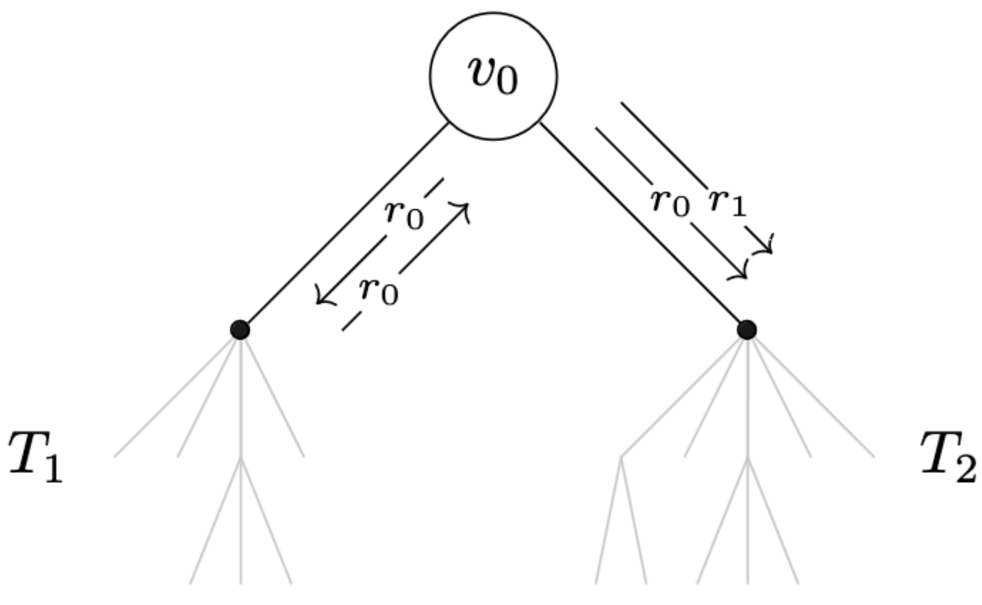
Step 1: When they meet at a point, say p (possibly on an edge), they evenly share the remaining energy.

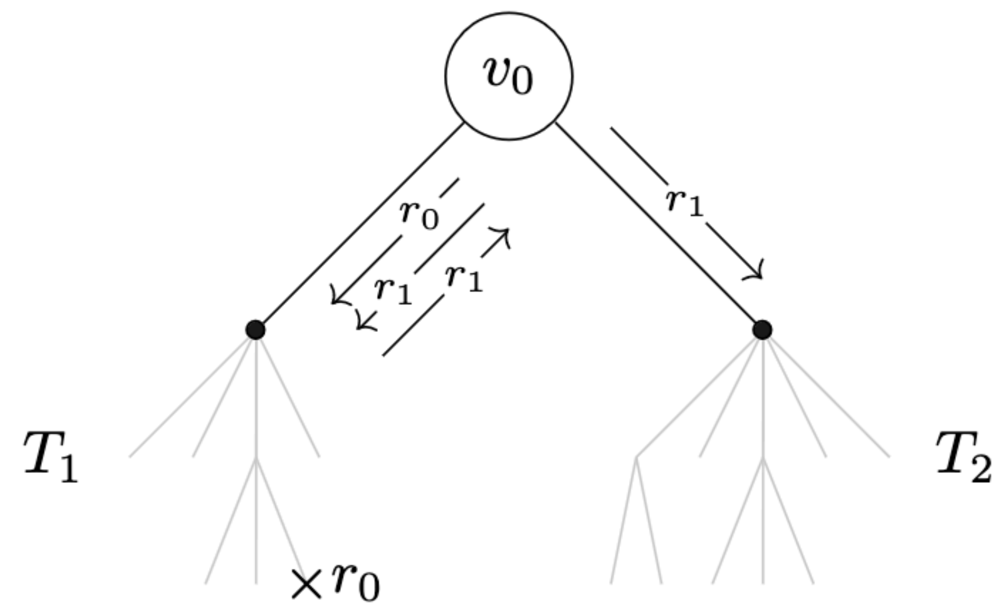
Step 2 Agent r_0 (resp. r_1) performs the following sequence of moves: move to the nearest node v_i of T_i; traverse T_i along a Eulerian tour of T_i in the clockwise direction (resp. the counter-clockwise direction) until the agent *(i)* meets the other, or *(ii)* completes the Eulerian tour traversal without meeting the other; move toward p until the agent meets the other in case of *ii)* if $i < k$; If $i < k$, set p be the meeting point, set $i = i + 1$, and continue from **Step 1**.

▶ **Theorem 12.** *If the agents start at the center of an unweighted star of size $\Delta + 1$, then the total energy consumption of any algorithm cannot be in $2\Delta + 2\log(o(\Delta))$.*

Trees

$$ct_2 \ : \ (en_0 \geq x) \wedge (en_1 \geq x) \wedge (en_0 + en_1 \geq 2W + d + x)$$





► **Theorem 13.** *There exists an infinite family of trees such that the required total energy is at least* $2W + \frac{d}{2} - 3$